DIGITAL SUPPORT OF MATHEMATICAL MODELLING: THE ROLE OF HINTS AND FEEDBACK IN MATHCITYMAP

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Abstract. Mathematical modelling addresses numerous demands of modern mathematics education: The learning of mathematics is directly linked to reality and students learn impressively what they can use mathematics for. At the same time, these tasks are very challenging for students, especially when the tasks need to be structured and validated without the immediate help of a teacher. Modern approaches, such as MathCityMap, pursue the support of modelling activities with digital technologies. In this paper, the potential of MathCityMap features like hints, solution validation, and sample solution are explored in modelling tasks in different contexts. The results show that especially the solution validation feature makes clear that sometimes a second run of the modeling circle may be necessary. Furthermore, practical observations on hint usage emerge, which should be considered in future activities, i.e. in terms of modelling activities in the context of distance and online education.

Key words: Digital tools, MathCityMap, mathematical modelling.

MATHEMATICAL MODELLING WITH DIGITAL TOOLS

As one of the general mathematical competencies, mathematical modelling has found its place in the mathematics curriculum. Precisely because of its indispensable relation to reality, modelling distinguishes itself from the classical inner-mathematical problem and is thus intended to clarify the application relevance of mathematics (Blum & Leiß, 2007). Idealized, the modelling process is described as a cycle. The modelling problem usually derives from reality, i.e. a real situation. In order to work on this problem mathematically, students have to (1) understand the problem and (2) simplify it by choosing important information. By mathematizing the problem (3), students transfer it into the world of mathematics to afterwards work on it mathematically (4). The gained result is transferred back to reality by interpreting (5) where it is validated (6) and presented (7) thereafter (Blum & Leiß, 2007; cf. Figure 1).

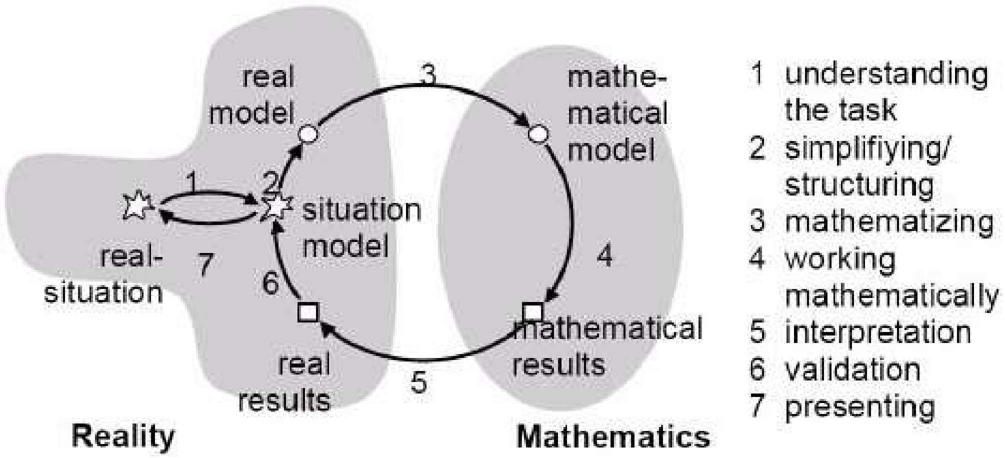

Figure 1: Modelling Cycle according to Blum & Leiß (2007).

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From empirical studies, it is known that mathematical modelling presents students with particular challenges in the following steps of the modelling process (Schukajlow, 2010):

- reading and understanding the task (1),
- recognizing the connection between facts of the situation and the mathematical solution structure (2 and 3),
- transforming mathematical structures (3), performing the arithmetic operations (4), and interpreting the results (5).

Additionally, empirical studies show that independent validation processes (6) rarely take place in students' modelling activities (Hankeln, 2020).

The challenges here described can lead to students being overwhelmed in solving the modelling problem satisfactorily (Niss & Blum, 2020). A support possibility of students in mathematical modelling is realized through the use of digital tools. Greefrath and Siller (2017) describe digital tools as a potential aid for teachers and learners, especially in the context of modelling problems. Depending on the tool and its purpose, different steps of mathematical modelling can be supported by digital tools, e.g. validation. By providing feedback on a given answer, the use of digital tools could promote and support these important mathematical activities. In general, there are two possible advantages of using digital tools in the modelling process: digital tools can be directly implemented into the steps of the modelling circle on the one hand, and on the other hand, they can elicit the execution of the individual steps in the modelling process.

FEATURES OF THE MATHCITYMAP SYSTEM

With MathCityMap, outdoor mathematical tasks can be created and solved in the context of a mathematics trail (Ludwig & Jesberg, 2015). While running a mathematics trail, students work in small groups and follow a route leading them to mathematics tasks being linked to real objects and situations.

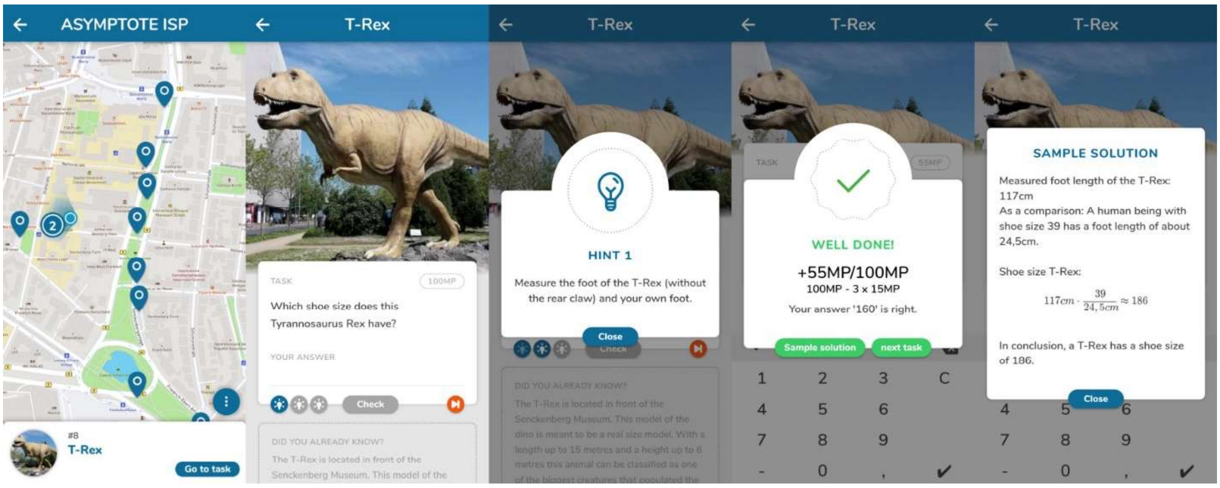

Figure 2: Screenshots from the MathCityMap app (a: Navigation, b: Task Formulation, c: Hint, d: Solution Validation, e: Sample Solution).

For mathematics trails in the educational purpose, a web portal for creating tasks and a smartphone app for walking mathematics trails are available. The smartphone app supports the students while they walk along a mathematical trail previously created in the portal and compensates the teacher's absence with its features when the students work outside the classroom. To do so, it shows the students' position and the location of the tasks (Figure 2a). A picture of the task object is available for clear identification. Furthermore, the tasks previously formulated by the teacher are displayed (Figure 2b), including up to three formulated hints (Figure 2c). In addition, the app provides direct feedback on the entered solution (Figure 2d) and displays a sample solution (Figure 2e). Since the Covid-19 pandemic, the scope of MathCityMap has been extended from outdoor mathematics to online education. During the Emergency Remote Teaching phase, it was used to compensate the absence of the teacher for distance education (cf. Barlovits et al., 2021) and therefore used in different contexts besides outdoor learning.

Independent of the context, two theoretical potentials of the MathCityMap features for mathematical modelling emerge (cf. Ludwig & Jablonski, 2021; Jablonski, 2023): On the one hand, the hints offer the possibility - depending on the formulation - to initiate and support individual steps of the modelling cycle. For example, support for structuring and simplifying the mathematical problem or help for mathematizing and the mathematical work can be implemented by the teacher through the hints feature beforehand. On the other hand, the validation option as described before, offers the possibility to initiate this step which can only rarely be found in the modelling activities. Especially in the area of modelling tasks, the solution check is conducted via an interval, which is determined in advance by various modelling steps and calculations and anticipating any deviations that could realistically occur. This kind of corrective feedback might draw students' attention to the fact that a new run of the modelling cycle might be necessary.

With these potentials only being on a theoretical level, the article aims at an empirical proof of the potentials of MathCityMap in mathematical modelling activities. Therefore, the research question *What is the role of the digital tool MathCityMap in students' modelling activities?*

METHODOLOGY

To answer the research question, the data from a qualitative study conducted in 2022 are partly taken for this article (for the whole study see Jablonski, 2023). In the total study, the different contexts of mathematical modelling were compared. For the research question of this article, only the sample with the focus on the digital tool MathCityMap is relevant. It comprises 10 students visiting the enrichment program *Junge Mathe-Adler Frankfurt* in grades 6-8. The students were divided into three groups. Each group worked on three tasks related to geometric objects (cf. Figure 3; from left to right): The height of the Body of Knowledge sculpture, the volume of the stone and the surface of the Rotazione sculpture.

One of the tasks was to determine the size of the *Body of Knowledge* sculpture if it were standing. The sculpture represents a seated person with legs drawn up. In order to provide a holistic view on mathematical modelling and its different contexts, three task settings were defined for each of the objects:

● Outside at the real object: The students solved the task outside directly at the real object. They had measuring materials with them as aids (see Figure 4a).

● Inside with photos: The students solved the task using a series of photos of the real object with a person as a possible reference size (see Figure 4b). In addition to the photos, they had a set square at their disposal for measuring sizes.

● Inside with 3D printing: To solve the task, the students were given a 3D representation of the real object, which had previously been printed to scale (see Figure 4c). A LEGO figure and a set square were provided as a reference size.

Figure 3: The task objects (a: Body of Knowledge, b: Stone, c: Sculpture).

Figure 4: Different Representations of the Body of Knowledge (a: Real Object, b: Photo, c: 3D Print).

The settings and objects were arranged systematically according to a Latin Square Design: Group A solved the Body of Knowledge with the 3D Print, the Stone at the Real Object and the Sculpture with Photos. Group B in comparison solved the Body of Knowledge at the Real Object, the Stone with Photos and the Sculpture with the 3D Print. Finally, Group C solved the Body of Knowledge with Photos, the Stone with the 3D Print and the Sculpture at the Real Object.

While working on the tasks, the groups had a smartphone, which they used to access the tasks via the MathCityMap app. Hereby, they were able to use hints that should support the *Simplify and Structure* and *Mathematize* steps, e.g.,

- Hint 1: Look for a model that you can use to describe the Body of Knowledge. There are several models that could be considered and none will fit perfectly;
- Hint 2: You can work with proportions, for example;
- Hint 3: Think about the values you need for your model and how you can approximate them as accurately as possible.

While using the hints was optional for them, the groups were explicitly asked to have their result validated by the app. After solving a task correctly or giving up, the students were able to view the sample solution with a possible solution of the modelling task.

During the processing of all tasks, the groups were accompanied by a student assistant who filmed the interactions. As the student assistant gave not content-related advice, the situation resembled a modelling process without the teacher being present as it happens during a math trail or in distance education. The video interactions were transcribed, coded deductively using the modelling steps according to Blum and Leiss (2007) despite Presenting (which was not relevant in the material) and visualized with activity diagrams (Ärlebäck & Albarracín, 2019). Besides, a qualitative content analysis according to Mayring (2000) was used to extract the main activities being related to the MathCityMap app. In addition to the video recordings and the written records of these groups, the app activities of the groups were recorded using MathCityMap's Digital Classroom. In an exported Excel list, the group's events (retrieval of hints, solution input) were made available with a timestamp so that they can be directly associated with the video recordings. Accordingly, MathCityMap events were labeled in the representation of modelling activities.

In a questionnaire that was given to the students after each modelling task, five items related to hint use and feedback on the solution provided by the app were given. The students were asked to give their agreement to the five items on a scale from *0: Not agree* at all to *4: Totally agree*. Since hint use in particular was optional and incorrect feedback also did not occur for all groups, *Not Used* was added to the scale.

- MCM_1: The hints helped me to structure the solution process.
- MCM_2: The hints helped me identify what math I needed to solve the problem.
- MCM_3: The feedback on the result allowed me to evaluate my solution as correct/wrong.
- MCM_4: In case of wrong feedback, the app made me check and improve my solution.
- MCM_5: I would like to solve tasks with the app more often.

RESULTS

The three groups took 21 hints. A total of 39 solution inputs were validated for the nine tasks. Finally, six tasks were validated as correct using the previously determined solution interval. In addition, the sample solution was consulted after four modelling processes – once after a correct solution, three times after multiple incorrect entries.

Figure 5 shows the activity diagram of one group solving the three tasks in the different contexts. The analysis of the diagram is done as an example for the task that a group worked on inside the classroom with photographs (second task). The group started to understand the modelling task, continued with structuring and simplifying the real situation and mathematized the real model. After a short period of mathematical work, the group entered their first result in the app which was validated as wrong. Based hereon, the students continued to validate their result and procedure and started a second attempt to solve the task properly. They rethought the steps of mathematizing, structuring and simplifying and working mathematically. Afterwards, they checked a second result in the app which was again rejected. Afterwards, they took the first hint and managed to proceed the task with a correct result through validation and identifying a previously made mistake.

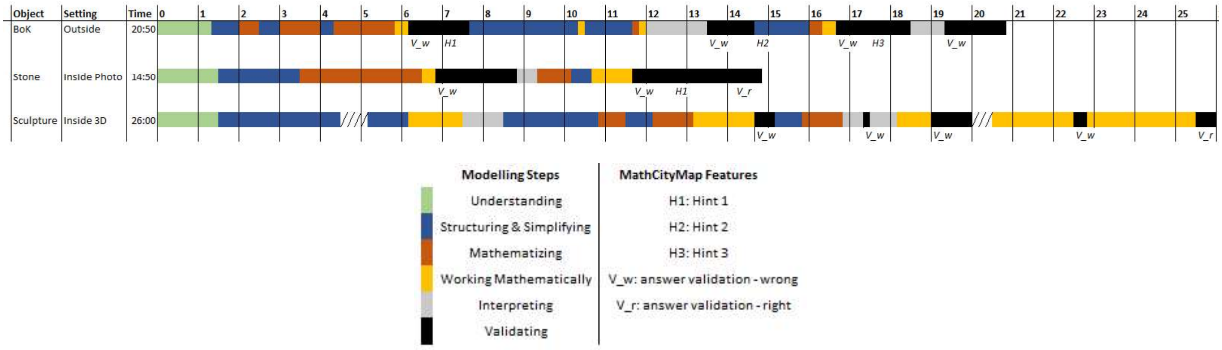

Figure 5: Example of an Activity Diagram visualizing a Modelling Process.

With respect to hint usage, the following observations can be formulated from this and the other two groups' diagrams: The hints were generally used in the course of an *on-going* modelling process – for most groups, the hints were only retrieved after an initial solution had been evaluated as "wrong" by the app. Thus, for this sample, the hints did not serve too much to initiate specific modelling steps, but to re-discuss and revise them. Based on the qualitative content analysis of the group interactions, the following support can be described in relation to the modelling steps:

- Simplify and Structure (2): In particular, hint 1 guided the modelling step and led to (re)discussing the choice of a real model considering inaccuracies.
- Mathematize (3): Through hints 2 and 3, aspects of mathematizing were addressed. Since these hints were taken only after prior mathematizing, they led in this phase to reconsidering and, if necessary, adjusting the data selection.
- Mathematical Work (4): In comparison, the hints were rarely used in the step of mathematical working. Since the hints did not directly fit to this step due to their wording, the students did not directly refer to the hints in their further work.
- Validation (6): The hints were used immediately after the app validation to assess why the entered result was evaluated as "wrong" (see the example in Figure 5).

The solution validation by the app was in all cases directly related to the modelling step of validation (6). In one case, a solution was validated as correct directly after the first input, which did not lead to any further action on the part of the students. In the other eight modelling processes, a solution was validated as "incorrect" at least once. After the first incorrect entry, the feedback from the app led all groups to start a second run of the modelling cycle – usually with the Simplify and Structure or Mathematize step. If there was another incorrect entry, sometimes a third attempt was started, in which the students checked their steps and changed them if necessary. If, despite repeated attempts, the app still validated the solution as "wrong," the sample solution was used in this case to complete the validation step. In these cases, the groups used the sample solution to compare their approach to an exemplar model and to identify potential problems.

For the questionnaire items, the following data can be reported: The students perceive the hints as rarely useful (M_{MCM_1} = 1.2; M_{MCM_2} = 1.1). In contrast, the evaluation tool of the app was perceived more positively (M_{MCM_3} = 3.1; M_{MCM_4} = 2.5). The wish for future use of the MathCityMap app reaches a mean value of 2.8.

DISCUSSION

With the presented results, it is possible to reflect on the question of MathCityMap's role in students' modelling activities. The students of this sample used all three features provided by the app: hints, solution validation and sample solution. The hints were mainly not used in the originally intended way: Whereas the hints should initiate modelling steps, the students mainly searched for support only after the first run through the modelling cycle and a negative solution validation. This is also reflected in the students' perception of the hints in terms of their guidance and usefulness. Therefore, it makes sense to formulate the hints in such a way that they do not initiate but support the modelling steps precisely, e.g. proposing useful simplifications or appropriate models.

For the solution feedback, it can be concluded that this feature helps students in validating their work – a step that usually does not take place independently on the side of the students. Through the app validation, the students started a new attempt to solve the task and revised problematic modelling steps from their first trial. This observation can also be linked to the agreement of the students to the corresponding items. For the sample solution, potential can be seen in terms of a complete and explaining feedback instrument. Whereas the automatic feedback can be valued as a meaningful feature in the context of the modelling cycle, the sample solutions play a comparable minor role. It was only used after inserting multiple wrong results. It shows that the students completed the task with the app giving a "correct" feedback and did not show additional interest in different modelling procedures. Therefore, the app can serve as a feedback instrument, but should not replace the discussion of different modelling strategies thereafter.

Concerning the different modelling contexts, the made observations show no relevant differences in the app usage dependent on the context. The groups used the app similarly in the different contexts. The predominantly positive attitude to future uses of the app, allows the proposal to include corrective feedback through digital tools in mathematical modelling activities as it happens in MathCityMap for outdoor education and modelling contexts inside the classroom. Even with their teacher being absent, the digital tool can help students to understand the modelling process as a process that has to be revised and improved before a result is acceptable. In addition, it helps them to understand their result as valid and might lead to more independent validation activities in future modelling activities. This hypothesis as well as the potential of improved and differentiated hints are to be examined in future research and in research with a special focus on digital tool in distance modelling activities.

In this future research, two limitations of the presented study should be considered in more detail. The first is the choice of sample, i.e. gifted and interested students. It might be that the results need adaptations when being transferred to more heterogeneous groups. Mathematically gifted children might bring special (problem solving) skills that could support their work in the modelling tasks as well. In addition, the sample of this study should be extended in terms of the quantity of observed students.

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