AN EXPERIENCE WITH AUGMENTED MATH TRAILS AND SERVICE-LEARNING IN INITIAL TEACHER TRAINING

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Abstract. *We present the ongoing project developed at the Universidad Autónoma de Madrid (Spain) using math trails in Initial Teacher Training with the aim that they can incorporate this educational resource into their future professional career. The learning experience is an integrated project focused on the creation of math trails by prospective students as a valuable outdoor problem-posing activity. In addition, it includes a two-step evaluation process that, first, allows prospective teachers to assess each other's routes, and secondly, they are tested by students as a service-learning activity. The math trails used in the project include mobile technology as an enriching element, combining the experience of using MathCityMap and the Augmented Reality tools of GeoGebra within the same project.*

Key words: Augmented reality, initial teacher training, math trails.

INTRODUCTION

Math trails can be defined as "a walk to discover mathematics" that can be almost everywhere (at the street, inside buildings, in parks, museums, zoos, etc), with marked stops "where walkers formulate, discuss, and solve interesting mathematical problems" (Shoaf et al., 2004). They are very flexible and creative educational resources. They can take place in urban, natural, or artistic settings, and can be adapted not only to any educational level, but also for tourism or divulgation purposes.

It is a form of outdoor learning which connects mathematics inside and outside the classroom. Students must solve mathematical tasks along the route, so it is a problem-solving activity in a real context which highlights the presence and relevance of mathematics in common objects and daily situations. It also gives them the opportunity to have an active and collaborative mathematical learning experience in a more relaxing atmosphere. Math trails are meant to use the spaces around us as an educational space, so they have a great potential as an interdisciplinary tool. Therefore, the inclusion of math presents as an opportunity to develop their mathematical competences and to provide future teachers with an experience of creative and collaborative work related to mathematics that they will be able to incorporate in their professional future (Barbosa & Vale, 2016; Moffett, 2011).

This paper presents a learning experience at Universidad Autónoma de Madrid with pre-service teachers where they use and design math trails with mobile technologies. The main goal is to involve prospective teachers in an integrated project with the aim that they can incorporate this educational resource into their future professional career. In addition, it gives students an opportunity to provide a service to the society with their learning in an authentic setting.

For the appropriate integration of technology in educational activities, the theoretical framework that stands up is the TPACK (Technological Pedagogical Content Knowledge)

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developed by Mishra and Koehler (2006). There have been several previous integrated approaches to enhance pre-service teachers' TPACK such as the SQD model in (Tondeur et al., 2020). In this direction, the present paper follows a teacher training strategy divided in four stages: (1) experience, (2) design, (3) peer-review, and (4) service-learning.

This experimental inclusion of innovative tools in Mathematics Education belongs to an ongoing research project. During all the stages, we have asked our students to fill out questionnaires, quantitative and qualitative, before and after the experience. In addition, they hand in the group dossiers with the problem-solving strategies.

THEORETICAL FRAMEWORK

Math trails in initial teacher training

The experimentation and creation of math trails is an enriching activity for prospective teachers (Barbosa & Vale, 2020; Haas et al., 2021; Martínez-Jiménez et al., 2022b). It is presented as an open challenge, becoming a mathematical creative learning process which requires the design of contextualized tasks for their future pupils. It provides them with a practical activity which shifts from usual academic settings to real-life mathematical situations. It constitutes an opportunity to develop problem-solving and problem-posing skills in prospective teachers, and to provide them with a collaborative mathematical experience that they can incorporate in their professional future (Figure 1).

Figure 1: Prospective students at UAM in a math trail (left) and creating a mathematical task (right).

The aim is to engage them with this outdoor learning process and give them the confidence to include math trails in their professional future. It is a collaborative math experience new for most of them, learning how to look at our surroundings with "mathematical eyes".

Use of mobile technology in mathematical education

The use of mobile technology has been successfully implemented in several outdoor math learning proposals. It gives more flexibility and autonomy during the activity and gives teachers the possibility to give feedback and interact with students (Cahyono & Ludwig, 2019; Wijers et al., 2010). In relation to math trails, in this paper we consider two: MathCityMap and the Augmented Reality (AR) tools of GeoGebra.

MathCityMap

Developed at the Goethe Universität Frankfurt in 2012, it is an easy-to-use online platform (https://mathcitymap.eu/) and a mobile App where teachers can create, share, and carry out math trails. Some of the key features that makes it a useful tool to perform and create math trails are the possibility to create *digital classrooms* to carry out trails with students, the GPS geolocation of the trail, the interaction between students and teachers via chat, the inclusion of gamification features, hints to solve the problems and immediate feedback after the task is concluded (Ludwig & Jablonski, 2021).

Augmented Reality

One of the applications of AR tools into educational settings is its introduction in outdoor activities such as math trails. This technology adds an extra "mathematical layer" to the location or to the real object considered in a task, increasing the variety of possible math problems and ways to solve them. Works in this direction can be found in Botana et al. (2020) with GeoGebra AR and Cahyono et al. (2020) with Secondary Education students (Figure 2).

Figure 2: Uses of Augmented Reality in Botana et al. (2020) (left) and Cahyono et al. (2020) (right).

THE MATH TRAILS PROJECT AT UNIVERSIDAD AUTÓNOMA DE MADRID (UAM)

Ongoing since the academic course 2019-2020, we have been using math trails at courses of *Mathematics and its didactics* at undergraduate and master's degrees in education at UAM. The degrees involved are the Early Childhood Education Degree (3-6 years old), Primary Education Degree (6-12 years old), Master in Secondary Education (12-18 years old) and Master in Innovation in Specific Didactics. In total, more than 600 prospective teachers have participated in the different stages of the project.

The main purpose of this project is the creation of mathematical routes by the future teachers (Stage 2). For this, a previous step of experiencing a mathematical route is necessary (Stage 1), which is done using MathCityMap. To complete the learning process, the trails created are tested and self-assessed by the students in a peer-review session (Stage 3). With the feedback provided, the math trails are redesigned and finally implemented with school students (Stage 4). Details about the 4 stages are the following:

- **Stage 1: Math trail experience**. Most of the students had never taken a math trail in their previous education, so the initial step is to experience a mathematical route. The students are divided in groups and join a *digital classroom* in the MathCityMap App to follow the trail. The gamification features are activated to keep track of the scores of the groups at every single task (Figure 3).

Figure 3: Image of the UAM Campus and location of mathematical tasks.

There are 5 pre-designed routes in MathCityMap around the UAM Campus, aimed for the different degrees involved:

Route 1. For students of the Early Childhood Education Degree (MCM code: 564103).

Route 2. For students of the Primary Education Degree (MCM code: 343382).

Route 3. For students of the Primary Education Degree in the optional course *Mathematics in Art and Nature*. The tasks of the trail are based on natural elements of the surroundings, taking students to the neighboring Monte de Valdelatas (MCM code: 127856).

Route 4. For students of the Master in Secondary Education taking the specialty in Mathematics (MCM code: 082917).

Route 5. Augmented Reality route designed for students of the Master in Secondary Education taking the specialty in Mathematics (MCM code: 133601).

In addition to the mobile app, every group has a dossier to write down their problem-solving strategies and calculations. In some tasks, mostly in Routes 1 and 2, students are asked to solve them in the dossier (the task is set up as an *Information Point* in the app). This simulates a task in a math trail without the use of technology, which is relevant for students in the Early Childhood and Primary Education degrees since their future students will not (usually) have access to mobile devices (Figure 4).

Figure 4: Students during a math trail with MathCityMap at the UAM Campus.

- **Stage 2. Creation of math trails.** Prospective teachers design in groups math trails aimed for their future pupils. The work at this stage can follow two options: (1) students create their math trails at any location around the city (streets, nearby schools, etc.), or (2) students create their math trails at the UAM Campus.

Figure 5: Math trail created by a prospective teacher in Madrid.

In Option 1, there is more freedom to choose the location of the route and the relationship between mathematics and day-to-day situations is better solved. In Option 2, the proximity to the University makes easier to review and test the routes created (Figure 5).

The use of MathCityMap is usually optional, although in some cases it is mandatory (for example, when the groups create their trails at the UAM Campus or with students of courses of *Technology in Mathematics Education)*.

- **Stage 3. Peer-review.** When the students create their math trails at the UAM Campus it is possible to do a review session where groups of students (peers) carry out each other's math trails. The MathCityMap portal is very helpful during this process, since it gives the possibility to create groups for students to share and modify their tasks.

During the peer-review session, students evaluate the work of their partners filling out a rubric for every completed task, and a general questionnaire at the end of the trail.

- **Stage 4. Service-learning.** This final stage connects the learning process of the prospective teachers with their professional future and seeks to enrich their creative outdoor proposals with a service to their community. To this end, the trails designed by prospective teachers are tested with either schoolers (if the routes were created next to a school), or students from schools visiting the UAM (if they were located at UAM Campus).

The data collected in Stage 3 together with the teacher's evaluation serve to redesign the previous work and elaborate the final mathematical routes that will be put into practice with school students.

EXPERIMENTING WITH AUGMENTED REALITY

Prospective Secondary School teachers experienced a MathCityMap trail augmented with GeoGebra 3D applets. This experience was carried out by students of *Technology in Mathematics Education,* where they had previous training with GeoGebra. In addition, before doing the augmented math trail they had a pre-session with these tools. They were asked to model different wooden solids with GeoGebra and check if their model fitted the real 3D object (https://www.geogebra.org/m/s2kcquhj) (Figure 6).

Figure 6: Images of the modeling activity with Augmented Reality and GeoGebra.

With this activity students learn how to place and manipulate GeoGebra objects and to obtain the perfect adjustment of the 3D model. To do so, they need to make several test-redesign processes, thus going through modeling cycles during the activity.

Augmented Reality tasks in math trails

The *augmented math trail* requires two mobile apps: MathCityMap and GeoGebra 3D Calculator. Students are divided in groups and enter a digital classroom in MathCityMap where they can follow the route. At every task, there is a link to a GeoGebra applet with augmented data of the location or the object (a 3D model, hints, extra data, etc).

Once the applet is open with GeoGebra 3D Calculator the student can place the model into the reality. The objects in this outdoor setting are bigger than the pre-session models, so the difficulties in the placement of the 3D-models increase. Nevertheless, having the GeoGebra tools at their disposal was of great help for solving the tasks.

Some examples of possible uses of Augmented Reality in the outdoor setting of math trails include: (1) AR as a help for solving a task, (2) the use of AR as a dynamic model or (3) providing new or missing data of the object/location. Examples of this uses contained in the Route 5 in the UAM Campus are the following:

Example 1. AR as a helping device. In this type of task, the GeoGebra applet shows the 3D model of the object related to the mathematical problem. With GeoGebra check boxes, texts, and dynamic tools, like sliders, it is possible to highlight the key parts of the mathematical model or include measures and hints. Figure 7 shows the example of a task asking to calculate the volume of the recycling box, where the 3D model highlights the measures and details of its structure to help students in their calculations.

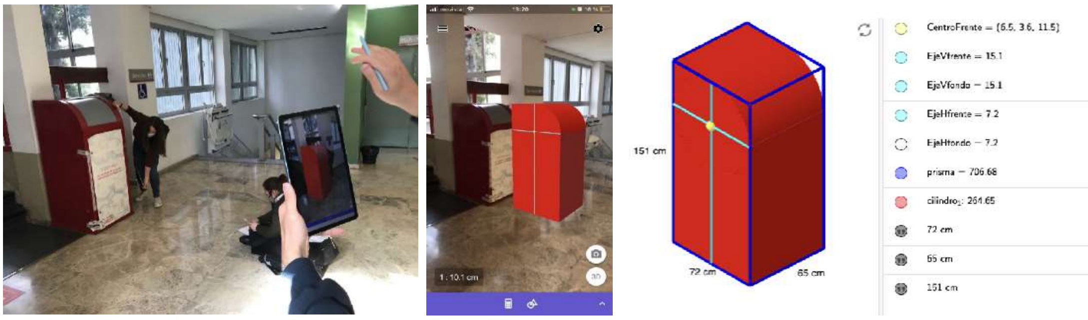

Figure 7: Students using the AR in a math trail and the GeoGebra model.

Example 2. AR as a dynamic model to solve tasks. Students can use the GeoGebra model to solve problems by placing the object in the location and looking at the data that GeoGebra displays. For example, it is possible to check volumes of flowerpots (Figure 8, left) or place stairs in an open space (Figure 8, right). In both cases, with the use of sliders the students can adjust the models to reality.

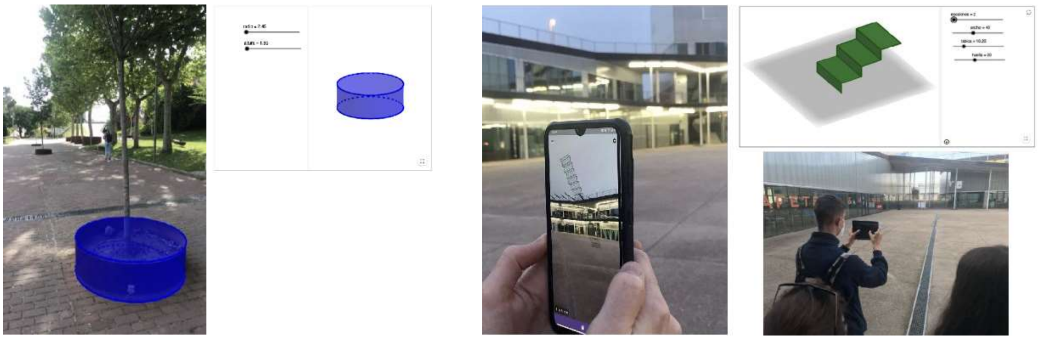

Figure 8: Augmented math trail tasks: *Which flowerpot is bigger?* (left); *How many steps do you need to reach the top of the building?* (right).

*Example 3. AR providing new or missing data to solve tasks***.** With Augmented Reality tools it is possible to enrich the mathematical content of the problem by including new or missing objects of the location. This is the case of the task shown in Figure 9, where students can see an extension of the tiles forming the floor of a sculpture at UAM Campus. The goal was to measure the amount of metal plates that the artist bought, before cutting them to make the final rectangle (shown in green).

Figure 9: Augmented task where students have the extended model of the floor.

CONCLUSIONS

Following the evidence given by the questionnaires filled in by students, in general terms, the project has been very positively evaluated. They showed positive attitudes towards math outdoor activities, and most of them found the inclusion of MathCityMap and GeoGebra engaging and motivating. For instance, the SIMS test (Guay et al., 2000) during the math trails showed high scores in the positive dimensions of motivation, while the negative dimension had low scores. Partial results in this line can be found in (Martínez-Jiménez et al., 2022a). Moreover, they considered the creation process of math trails meaningful for their professional future. Indeed, some of them have included math trails in their teaching, creating new routes near their schools.

In relation to the AR experience and its inclusion into math trails, the future Secondary School math teachers showed good responses in participation and motivation, as well as noticing room for improvements concerning the GeoGebra AR tool. For instance, they found it difficult to place the models into real objects of big size, mainly because of perspective misalignments and the objects do not remain fixed at the location.

During the augmented math trail, the use of AR which required more interaction with the GeoGebra calculation tools to solve the task were considered more engaging by the students, finding very attractive the idea of carrying out a math trail with such technology at their disposal. Nevertheless, most of the students appreciated the potential of AR in education but still see the implementation in the mathematics classroom far away, so further experimentation and research in this direction is required.

Finally, in relation to teacher training and the use of mobile technologies, it should be noted that although it provides prospective teachers with technological competence, it comes to limitations when putting into practice with their future pupils. Namely, in terms of access and use of these technologies and some constraints in the types of mathematical problems that can be posed.

Regarding Augmented Reality, the process of experimenting with its inclusion in mathematics activities should continue. Despite the advantages of GeoGebra being a very versatile software, the AR tool needs further development for its use with large objects outdoors. In addition, it would be appropriate to explore other software, as well as the potential of using markers and GPS positioning in math trails.

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