# BRINGING 3D PRINTING INTO STUDENT TEACHERS' MATHEMATICS EDUCATION

Tim Läufer and Matthias Ludwig
Goethe University, Frankfurt, Germany

***Abstract.*** 3D printing has been used in classes by teachers previously in several studies. In Germany, a lot of schools lack the funds or opportunities to use 3D printing in classes on a broad scale. Therefore, a seminar is in development and in the second iteration at Goethe University to train student teachers (educators) to create their own manipulatives for mathematics classes, where they worked intensely on mathematical and didactical aspects of their manipulatives. We present the underlying theory of 3D printing, and an ongoing teaching experience with student teachers using 3D printing for mathematics education. Additionally, students' expectations were evaluated to improve future iterations of the seminar. Some manipulatives from the seminar are presented.

*Key words: 3D printing, manipulatives, student teachers.*

## INTRODUCTION: 3D PRINTING IN MATHEMATICS CLASSES

There are certain topics in education that benefit greatly from examples that can be represented by three-dimensional objects, like the cubic formulae (Figure 1) in mathematics or sugar molecules in chemistry. This is especially interesting for natural sciences, where the physical world, real objects, schematic models, miniatures and similar can be scaled up or down and be held in one's hands, observed and investigated. These materials are called manipulatives (Larbi & Mavis, 2016). But not every object or example can be bought (for a reasonable price) or manufactured for a whole classroom without significant strain and effort by teachers and educators.

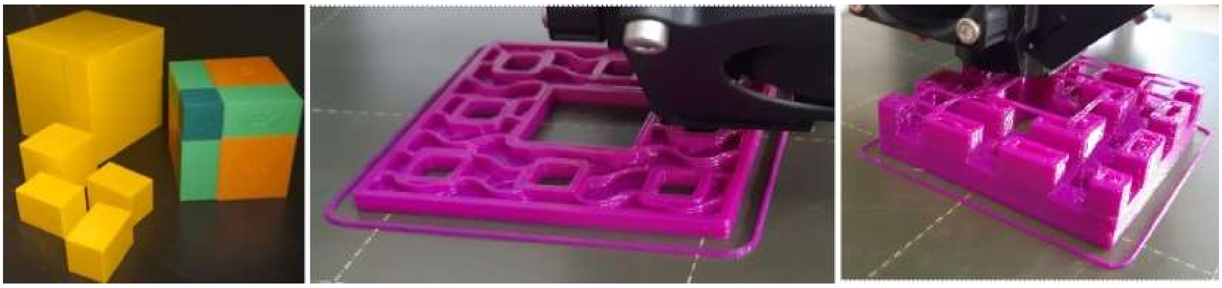

Figure 1: A student's manipulative (left), and the process of additive printing (center and right).

3D printing (3DP) is a practical way of manufacturing three-dimensional objects. Even though there are variations on how exactly, the most common one is additive manufacturing. A digital 3D model is sliced by a software into layers that then subsequently get printed with molten plastic onto a build plate, where every layer provides the needed support for the layers above. This technology was used first in an industrial context, where prototypes of certain objects could be manufactured quickly. Through the years 3DP has become more accessible to the public due to reasonable prices and is therefore now available to use in schools and education in general. (Gür, 2015)

Läufer, T., & Ludwig, M. (2023). Bringing 3D Printing into Student Teachers' Mathematics Education. In M. Ludwig, S. Barlovits, A. Caldeira, & A. Moura (Eds.), *Research On STEM Education in the Digital Age. Proceedings of the ROSEDA Conference* (pp. 49–56). WTM. https://doi.org/10.37626/GA9783959872522.0.06

It has been shown that integrating 3DP in STEM education impacts mathematical knowledge, spatial ability and technical skills, as well as attitudes towards mathematics, engagement and motivation positively (Kit Ng, Tsui, & Yuen, 2022). Pearson and Dubé (2022) found that 3DP is used to connect different subjects in K-12 education and engage students with challenging topics, but mainly when students have the chance given by their educators, school and curricula to use 3DP during lessons (Anđić et al., 2022; Ford & Minshall, 2019; Kit Ng et al., 2022; Pearson & Dubé, 2022). A lot of schools lack the technical requirements to make this happen: Be it missing funds or the high maintenance needs of printers; 3D printers may be difficult to obtain or keep intact and ready for printing (Bull et al., 2015); German schools' technical infrastructure and their progress lag behind the current needs for digital education (Lorenz et al., 2021) and there are several problems when using 3D Printers in schools, such as long print times, high maintenance needs, and high instructor effort, and low access to help when facing difficulties as an educator (Anđić et al., 2022; Bull et al., 2015; Ford & Minshall, 2019; Pearson & Dubé, 2022). Some teachers also feel that 3DP is not ready to be used in schools due to the aforementioned problems. (Ford & Minshall, 2019)

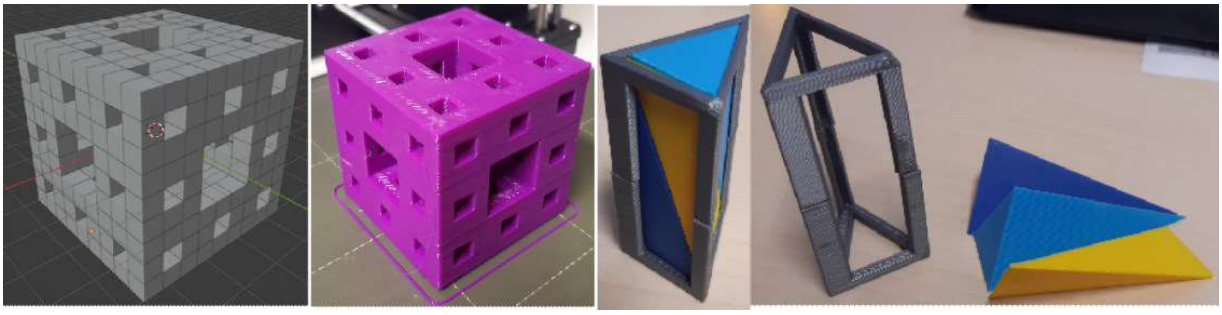

Figure 2: A 3D model of a Menger sponge (left), the model printed (2nd to left), and a manipulative of a prism that is "cut" into three equivalent pyramids (right).

Due to this a focus on using 3DP as an educator to manufacture manipulatives that aid learning is appropriate. This brings the benefit of reduced instructor effort during lessons, reduced cost due to fewer printers and materials needed, and less need for maintenance. For this, a seminar has been developed to teach educators (student teachers) how to use 3DP to enrich mathematical lessons, following Ford and Minshall (2019)'s identified need to include 3DP in pre-service teaching to allow teachers to use 3DP in their lessons. „The issue 'teaching the teachers' is a pressing one but one that appears to have been overlooked" (Ford & Minshall, 2019, p. 156). Teaching educators about 3DP and using 3DP to produce artefacts that aid learning are typical uses of 3DP in the education system (Ford & Minshall, 2019).

Anđić et al. (2022) have found that teachers may also see 3D-Modeling-Printing (3DMP) workflow as a trigger for themselves to design new tasks for students, new activities for their classroom, and "modernizing and adapting approaches and teaching methods" (Anđić et al., p. 8). In addition, it may be "a useful teaching tool for their professional development, as it increases their abilities to teach STEM together with their digital, communication, and evaluation skills" (Anđić et al., p. 12).

With 3DP, realizing mathematical objects is "easier than ever" (Asempapa & Love, 2021, p. 87). Objects like the "Integraph", a historic mathematical device that creates an antiderivative of a function graphically (Dilling, 2020), can be manufactured in a cost-efficient - and once familiar: comfortable - way, to allow a historical approach to integration in calculus. Other examples include edge models of special solids (Hoffart, 2019), and specific support materials for special needs students (Kalina, 2019). In one case abstract mathematical models such as a Möbius strip have been fabricated (Gür, 2015) to allow the physical investigation of abstract objects. In an example case presented in our seminar, we show a prism that has been sliced into three equivalent pyramids (2), allowing an enactive introduction to the volume of a pyramid. Fractals may also be addressed via a Menger sponge, which is an example used (Figure 2).

Through this and other materials, students approach the relations between three-dimensional bodies in an embodied, enactive, hands-on way (Bruner, 1966; Tall, 2013). 3DP, as an example, "can (also) be used as the basis for students' mathematical modelling experiences" (Asempapa & Love, 2021, p. 86) when creating models reflecting the real world. The use of 3DP in schools may also provide an effective method for exploring modeling in a more realistic context; and can be used as the basis for students' mathematical modeling experiences (Asempapa & Love, 2021)

A lot of models can be downloaded from a publicly available website like "Thingiverse" and then printed on the available printer in the school, although there is a lack of high-quality model databases for education - specifically for mathematics (Ford & Minshall, 2019). To not have educators rely on models that are not tailored for education, and to allow educators to innovate their educational materials, teaching them how to design and print their models is appropriate.

## THE STRUCTURE OF THE SEMINAR AND EXPERIENCES MADE

Following this approach, we have developed a practical, weekly seminar (2 x 45 min) at Goethe University where student teachers get introduced 3DP and develop their own didactic materials. The course spans a whole semester (12 weeks) and has been completed once. The experience was used to adapt the course for a second implementation, following a basic approach inspired by the design-based research approach (Cobb et al., 2003; Cobb et al., 2016; Seel, 2012).

| Phase | Duration | Content |
|---|---|---|
| **Input** | 5 weeks | In this Input-phase students get introductions to the topics of 3DP, the 3DMP workflow, didactics of physical models as well as the technical capabilities of the scripting library, mainly Basic 3D Bodies, Translations, Algorithms & Iteration, as well as constructive solid geometry - the way to create new objects by using Boolean operators like difference, intersect and union on two existing objects. |
| **Hands-on** | 5 weeks | During the Hands-on-Phase the students begin and continue working on their models with minimal intervention from the educators. Through this, the students have the freedom to design their materials and realize their ideas. Access to the 3D printer is available during and outside of the course hours, during which |

| | | |
|---|---|---|
| | | upcoming problems are addressed and models are discussed in one-on-one discussions. |
| **Presen-tation** | 2 weeks | The presentation phase, only consisting of two weeks, served as a frame for the student teachers to present their materials to their peers, answering questions and explaining the theoretical use in school. The didactic materials must be embedded in theoretical but realistic lessons and the state curriculum for mathematics. |

Table 1: Phases, their duration, and their content of the seminar at Goethe University on 3DP

The first time the seminar was held we developed a prototype version of a mathematical scripting library in the modelling and animation program Blender, which allows the students to use the programming language Python to be used to construct 3D models. Over the first semester, we saw that the students had trouble realizing what a 3D printer can be used for, which we have traced back to not letting the students get first-hand experience themselves very early on. This problem especially led to several situations where students explained an idea for a manipulative that wasn't printable in one part or it may have been difficult to realize with an additive 3D printer's restrictions, for example, overhangs where the printer would've needed to print in the air, or large objects which would have taken multiple days to print. They had a lot of ideas but the knowledge about the restrictions of a 3D printer wasn't known to them. Additionally, students had trouble starting with the three dimensions and designing three-dimensional objects. We also continuously added new features and worked on enhancing the scripting library to have more features such as more complicated objects like a wire frame or hollow cone.

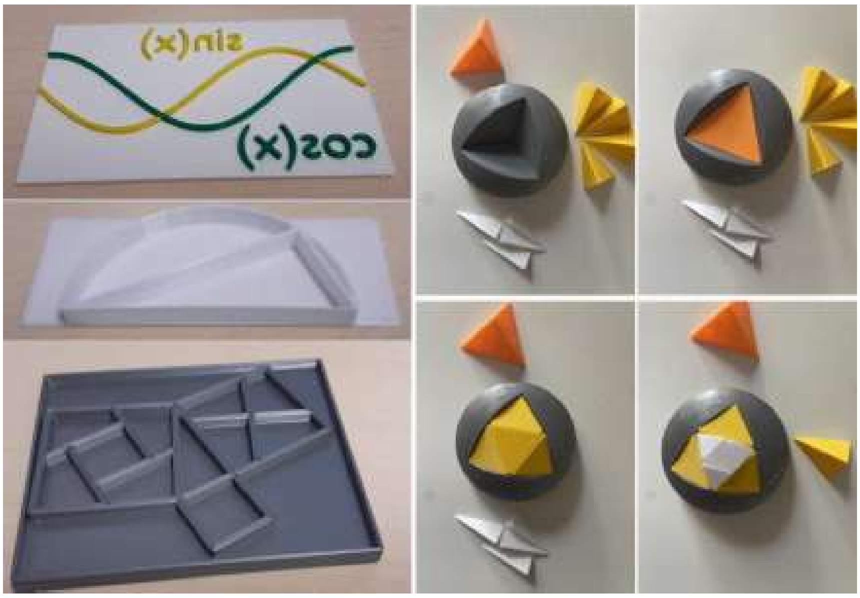

Figure 3: The "cookie stamps" example (left; sine/cosine, Thales' Theorem and Pythagoras' Theorem), and a students' manipulative (right) for introducing the approximation of a sphere. A "hole" in a sphere can be filled with pyramids whose top edges have the length of the circle's radius. Using more pyramids (yellow and white) approximates the sphere better than less (orange).

In the second iteration of the seminar, following the experiences of the first iteration, we started by reducing the number of dimensions the students worry about and began printing students' creations in the second session. The task was to design a cookie stamp with mathematical concepts or proofs (see Figure 3). There, students need to design a cookie stamp with 2D proofs or mathematical principles (i.e., Geometric series, Pythagoras' theorem, Thales' theorem, see Figure 3) that they are familiar with and whose individual components only feature circles, lines and points. These were printed in the second session and the students were very motivated. They could take their stamps home with them. The reasoning behind these manipulatives has been explained to the students and used to explain what a printer can and cannot print, after which students' ideas for manipulatives included the 3D printer's restrictions in the design itself. Explaining programming structures as if-else and loops have also helped students formulate their ideas more freely. The second iteration is being held in the winter semester of 2022/2023 until March 2023.

## STUDENTS EXPECTATIONS

To gain insight on the student teachers' needs and wants regarding this new technology we pose the question *What expectations do student teachers have towards a 3D printing seminar in mathematics education?* The answer to this question may lead to further insight on how a seminar to 3D printing may be designed to raise student motivation and gain knowledge on student needs towards such a seminar.

To answer this question there were two questionnaires via SosciSurvey that the students answered voluntarily. Through a shared, anonymous code in both questionnaires that can be recreated by knowledge only the students have, their before- and after answers could be combined. There they could react to their initial expectations and whether they were fulfilled.

In the first questionnaire, 11 students (3f, 6m, 1d), on average in the 9th semester (10 answers, mean = 9.1, std=1.37), aged 23-27 (10 answers, mean: 24,4, std: 1,17) reported on their previous knowledge regarding 3D printing. The second questionnaire was filled out by 10 of the 11 students.

Only one student had previously 3D printed their design, and one other had watched a video or read something about it. The students have not used any products commonly used in 3D design or printing (Blender, TinkerCad/AutoCad, SketchUp, Ultimaker Cura, PrusaSlicer, Paint 3D, POVRay, Maya, Fusion 360), except one who had experience with Blender and another who had experience who another, unmentioned program. The student teachers were asked to freely formulate their expectations towards a 3D-printing seminar ("What are your expectations of the seminar?"). These answers have been used to inductively create categories, the occurrences of which are shown in Figure 4. In the second questionnaire, their initial expectations given in the first questionnaire were given back to them with the question of assessing whether they were fulfilled. Only 6 students answered this item, with 5 of them agreeing that their expectations were fulfilled, and the last one only agreeing that it was instructive.

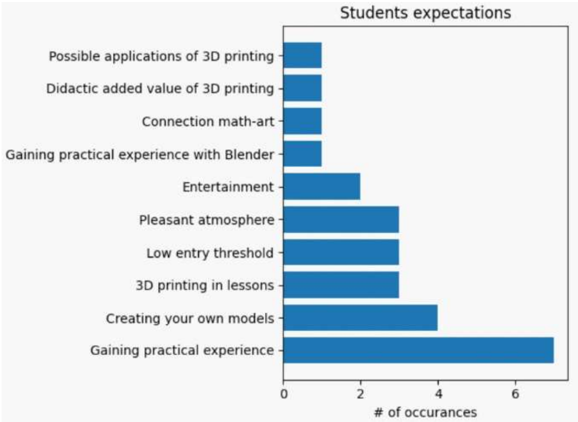


Figure 4: Students expectation categories and how often they occurred during the questionnaire before the semester began (left), and Students answers on 3D printing in math education (right, excerpt).

## STUDENTS MANIPULATIVES FROM THE FIRST SEMESTER

In this chapter, we highlight two didactical materials created by students. First, we have an approximation of a sphere (Figure 3), where a part of a sphere has been cut out and is replaced by a pyramid (orange) whose edges have the same length as the sphere's radius. It is directly visible that there is a significant portion of the sphere missing when substituting this part of the sphere with a pyramid. A substitution of the orange pyramid with four yellow pyramids, where each of them again has their edges' length be the spheres' radius shows, that the sphere is now "rounder". There is less of the sphere missing and we are approximating the sphere more appropriately. If we now substitute the center yellow pyramid with four white pyramids we further approximate the sphere. This model especially highlights students' needs for complex modelling, thus our decision to use script-based modelling is justified.

With this material teachers can introduce the concept of limits in a hands-on way. Students can see that by using more pyramids, each has its edge length be the spheres' radius, we approximate the sphere more and more. Of course, at some point, the 3Dmodel is insufficient in approximating further, but the concept can be grasped. An introduction to limits in the other direction - starting with a high number of pyramids approximating the sphere well - is also possible. A different model is shown in Figure 1 of the cubic formulas. The concept itself is not new, but the student has expanded the possibilities by adding colours, engravings, and distractor elements into the models. Through that, a larger differentiation can be achieved, as well as aid for special needs students through the engravings which can be felt by touch.

For higher-achieving students there is the puzzle with distractors, through which they need to figure out which elements to choose and which are not included in the formulas' results. It also lacks any kind of distinguishable color or engraving. Lower-achieving students can be aided by the color, engravings, and by taking away the distractors.

## DISCUSSION

The enactive, hands-on approach to student teachers designing and printing their manipulatives seems to be a very motivating experience, which was expected when comparing it to constructionism, where a construct serves as a representation of lessons learned (Papert, 1980). But due to the low number of students, a statistically significant effect has not been shown.

Using a script-based creation of 3D models may not be suited for everyone. We experienced that lower-achieving students struggle with algorithmic structures and basic expressions. A mix of both may be suitable depending on the students' needs and maybe even their orientation (higher, lower, special education). If they focus on less technically complex and more didactically complex objects, options like TinkerCad are very well suited to create didactically rich 3D models. In a hands-on way, students can deepen their knowledge in didactics regardless of their orientation (primary, upper/lower secondary, special needs) but applying it to their model is motivating and may lead to long-lasting learning. This has been shown in other research too (Ford & Minshall, 2019).

The freedom of the students to create what they want - within reason - has produced interesting and thoughtful models as well as models where mathematical content is hardly found. A constant discussion about what the goal is: that real students may use this, "so what should they do and learn?" helps guide the students to focus on the necessary.

We come to the same conclusions and experiences supported by various research (Ford & Minshall, 2019; Kit Ng et al., 2022; Pearson & Dubé, 2022) that 3DP is a valuable tool that can be used in education and may even help teachers further develop their professional acting (Anđić et al., 2022). We especially want to highlight the points that schools may lack the funds, teachers may lack the time to fully use 3DP in-lesson, or maintenance may be too high, which is why a focus on educators creating their manipulatives via 3DMP is very applicable and, proven by the presented seminar, possible even in the beginning of teacher education, namely universities.

Further questions may be what types of models may be created by either a script-based or a graphical way of creating 3D models and what the pros and cons are, or how student teachers' didactical and mathematical is fostered using 3DMP.

## REFERENCES

Anđić, B., Ulbrich, E., Dana-Picard, T., Cvjetićanin, S., Petrović, F., Lavicza, Z., & Maričić, M. (2023). A phenomenography study of STEM teachers' conceptions of using three-dimensional modeling and printing (3DMP) in teaching. *Journal of Science Education and Technology*, *32*(1), 45–60.

Asempapa, R. S., & Love, T. S. (2021). Teaching math modeling through 3D-printing: Examining the influence of an integrative professional development. *School Science and Mathematics*, *121*(2), 85–95.

Bruner, J. S. (1966). *Toward a theory of instruction* (vol. 3). Belknap Press.

Bull, G., Haj-Hariri, H., Atkins, R., & Moran, P. (2015). An Educational Framework for Digital Manufacturing in Schools. *3D Printing and Additive Manufacturing*, *2*(2), 42–49.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design Experiments in Educational Research. *Educational Researcher*, *32*(1), 9–13.

Cobb, P., Jackson, K., & Dunlap, C. (2016). Design Research: An Analysis and Critique. In L. D. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (pp. 481–503). Routledge.

Dilling, F. (2020). Qualitative Zugänge zur Integralrechnung durch Einsatz der 3D-Druck-Technologie. In G. Pinkernell & F. Schacht (Eds.), *Digitale Kompetenzen und curriculare Konsequenzen: Herbsttagung vom 27. bis 28. September 2019 an der Pädagogischen Hochschule Heidelberg* (pp. 57–68). Verlag Franzbecker.

Ford, S., & Minshall, T. (2019). Where and how 3D printing is used in teaching and education. *Additive Manufacturing*, *25*, 131–150.

Gür, Y. (2015). Digital Fabrication of Mathematical Models via Low-Cost 3D FDM Desktop Printer. *Acta Physica Polonica a*, *128*(2B), B-100–B-103.

Hoffart, E. (2019). Kantenmodelle mal anders: Merkmale und Eigenschaften des Würfels im Kontext der 3D-Druck-Technologie. m*athematik lehren*, (218), 13–16.

Kalina, U. (2019). Mit 3D-Druck Aufgaben (be)greifbar machen: Material für inklusiven Unterricht erstellen. *mathematik lehren*. (218), 21–22.

Kit Ng, D. T., Tsui, M. F., & Yuen, M. (2022). Exploring the use of 3D printing in mathematics education: A scoping review. *Asian Journal for Mathematics Education*, *1*(3), 338–358.

Larbi, E., & Mavis, O. (2016). The Use of Manipulatives in Mathematics Education. *Journal of Education and Practice*, *36*(7), 53–61.

Lorenz, R., Yotyodying, S., Eickelmann, B., & Endberg, M. (2021). *Schule digital – der Länderindikator 2021 Erste Ergebnisse und Analysen im Bundesländervergleich*. Waxmann.

Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas*. Basic Books.

Pearson, H. A., & Dubé, A. K. (2022). 3D printing as an educational technology: theoretical perspectives, learning outcomes, and recommendations for practice. *Education and Information Technologies*, *27*(3), 3037–3064.

Seel, N. M. (2012). Design Experiments. In N. M. Seel (Ed.), *Encyclopedia of the Sciences of Learning* (pp. 925–928). Springer.

Tall, D. O. (2013). *How humans learn to think mathematically: Exploring the three worlds of mathematics*. Cambridge University Press.