ENACTIVE LEARNING IN MATHEMATICS AT HOME - THEORETICAL FRAMEWORK

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Abstract. The project "Enactive Learning in Mathematics at Home (EnLeMaH)" aims to promote enactive work of pupils in the area of functional relationships. This paper establishes the theoretical foundations with respect to an understanding of 'enactive learning', learning fundamental, and experimental work in mathematics. This paper is thus the theoretical basis for a workshop on enactive working.

Key words: Bruner, enactive work, experiments.

INTRODUCTION

Introducing an enactive approach to Mathematics teaching helps pupils build a mental network to understand mathematical concepts and relations and how they can use mathematics in their daily lives. Enactive learning means having handmade activities, experiments and concrete handling with material to enter new mathematical topics, have mental representations of mathematical content and discover mathematical relations. Consequently, enactive methodologies help to increase the understanding and the attractiveness of mathematics and, to a broader extent, contribute to reduce underperformance. Nevertheless, the adoption of an enactive approach to mathematics is based on two main preconditions or premises: On the one hand, teachers need to acquire and be equipped with the adequate pedagogical skills to implement this methodology, particularly when it concerns to its applicability to the context of digital education and training. On the other hand, enactive materials can be hard to obtain in the current context. The project "Enactive Learning in Mathematics at Home (EnLeMaH)" promote the adoption of innovative digital pedagogical competencies for mathematics school teachers, which will enable them to develop the knowledge and skills to: (1) Implement an enactive teaching & learning methodology adapted to the context of digital education; (2) Guide pupils in creating, using household supplies, enactive materials that support their learning processes, with a special focus on Mathematics learning in the field of functions.

THEORY OF COGNITIVE GROWTH BY JEROME S. BRUNER

It is fruitful, I think, to distinguish three systems of processing information by which human beings construct models of their word: through action, through imagery, and through language (Bruner 1966, p.1).

As Bruner stated above, individuals represent their learning and the world in which they live through action if they cannot do so using images or words. He assumed about learning that any subject could be taught at any stage of development in such a way that it‘s cognitive abilities were met. To learn a more highly skilled activity, it has to „be decomposed into

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simpler components, each of which can be carried out by a less skilled operator“ (Bruner 1966, p. 2). Representations are the product of a system of coding and processing past experiences. Hence, he introduced a model including three modes of representation as included in Table 1). He believed that people represent their knowledge in those three ways. Modes of representations “are not structures, but rather involve different forms of cognitive processing” (Schunk 2012, p.457).

<table>
<tr><th colspan="2">Name of the mode of representation</th><th>Description</th><th>Examples in mathematics classes</th></tr>
<tr><td colspan="2">Enactive mode of representation</td><td>Suggests that individuals represent their learning and the world in which they live through action A pupil best understands their environment by interacting with the objects around him</td><td>
Using material to represent a mathematical concept.</td></tr>
<tr><td colspan="2">Iconic mode of representation</td><td>Summarizes events of precepts and of images, by the spatial, tempo-ral, and qualitative structures of the perceptual field and their transformed images.</td><td>Using images (e.g. pictures of the (mathematical) situation, graphs) to represent a mathematical concept</td></tr>
<tr><td rowspan="2">Symbolic mode of represent-tation</td><td>Verbal-symbolic</td><td>Each word has a fixed relation to something it represents</td><td>Using (actually spoken) word to represent a mathematical concept</td></tr>
<tr><td>Non-verbal-symbolic</td><td>Each symbol has a fixed relation to something it represents</td><td>Using written sentences and mathematical symbols (e.g. equations) to represent a mathematical concept</td></tr>
</table>

Table 1: Modes of representation.

For enactive learning, these modes of representation correspond in the learning process (Figure 1). Enactive and iconic representations can yield symbolic representations and vice versa: enactive or iconic representations can be derived from symbolic representations.

By this, the three modes of representation deal with a central theoretical aspect for the EnLeMaH-project: For the understanding and the creation of enactive learning activities the separation between different modes are basic. In the next chapter, we take a deeper look at aspects for designing enactive learning.

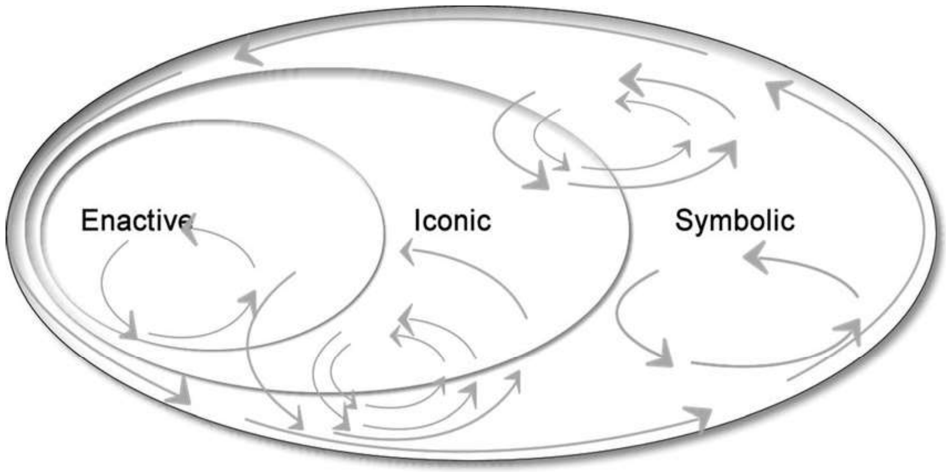

Figure 1: Enactive, iconic and symbolic as nested, co-implicated and simultaneous (Francis, Khan & David, 2016, p. 8).

BIOLOGICAL BASES FOR ENACTIVE LEARNING

According to Di Paolo (2018), the term enactive was used prior to the biological bases that shape the theory today. For example, Bruner (1966) used the term enactive to establish a relationship between representations and bodily aspects belonging to a person's lived experience. Currently the meaning of enactivism is based on the works started by the biologist Francisco Varela and on the works carried out jointly with Maturana (1987). Today this theoretical perspective continues its development by various groups of researchers who are focused on different areas of study (cf. Brown, 2015)

Varela, Thompson and Rosch (1991) used the words 'enaction' and 'enactive' to describe the non-representation list view of cognition they set out, cognition as "embodied action" (Varela et al., 1991, p. 172). This refers to two important points: (1) perception consists in perceptually guided action and (2) cognitive structures emerge from the recurrent sensorimotor patterns that enable action to be perceptually guided. Therefore to understand what enaction is, one must understand what perception is. It is important to note that perception is sometimes seen as a passive process (e.g. when light enters your eyes and you are able to create an image), but in enactivism, perception is an active process, and without action there is no perception. This active process is determined by the structure of the perceiver, for example: how a bird perceives a certain situation is very different from how a person might perceive it. Thus, the organization is understood as the relationships that must exist between the components of something to be recognized as a member of a specific class, and structures of something is understood as the components and relationships that specifically constitute a particular unit that its organization carries out (Lozano, 2014). This occurs in the particular mode of organization which is called autopoiesis. Therefore, an autopoietic system is one that despite being constantly changing and producing new reference systems, the result will always be the same producer. According to Matura (1987), the problem would be how to handle the problem of change of structure and show how an organism that exists in an environment and that operates adequately according to its needs, can undergo continuous structural changes even if the environment is changing. So, this could be an approximation to the problem of teaching mathematics, a learning person is a system that internally organizes itself at every moment. So, every time a stimulus reaches it, (for example a mathematical symbol), it is immediately incorporated into the person's structure, into her being.

According to Lozano (2014), when living beings interact with the environment where other living beings are included and there is a recurring interaction between two systems, then both will change in a similar way. From this perspective, we could say that when a learner interacts repeatedly with her teacher and with the other learners, together they will create a history of interactions. Therefore, the structure of all those who are participating in these classes can change in a similar way, creating new forms of communication and work. If this does not happen, then the structural changes do not lead to adaptation to the environment. Lozano (2014), presents a clear example for this: if a learner repeatedly fails math tests, in a certain context this could mean that the learner changes the learning group he/she is in.

Something important to mention, is that the world is not something that is given to us, but something that we relate to by moving, touching, breathing, and eating, this is what Maturana and Varela called cognition as enactive (Maturana and Varela, 1992). So, enactivism indicates that our mental activity (thoughts, images, emotions) is rooted in the actions we carry out with and through our bodies. The enactivism point of view, learning arises as we actively interact with the environment, so it cannot be thought of as absorption of information and cognition is not a phenomenon that arises within the head or body of a single individual, but arises from continuous interactions with the environment, which in turn is modified by these. In our case, society and culture are part of our environment as human beings.

This concept of enactivism from a biological perspective invites us to reflect on the importance of the type of activities chosen to address a mathematical learning objective. In general, there are many materials available to us, but we need to take into account the context in which our learners are developing, the nature of the environment and the type of structure that makes them up. This means that we must try to create task models that are appropriate to the level of ourlearners and at the same time use materials that allow them to use enactive actions to capture new learning.

EXPERIMENTS AS PART OF ENACTIVE WORK

An experiment is a scientific method designed to gather information. It is used both at school and at university and also in several subjects. Experiments in mathematics education are used in different contexts, especially the difference between experiments in mathematics and other subjects is emphasised (Artigue & Blomhøj, 2013). For our approach here, we focus in particular on the enactive aspects in mathematical experiments. For this purpose, the goals of mathematical experimentation are first explained and the individual steps are derived in the second step.

The method "experiment" – differencies between subjects

Following Kirchner et al., there are different purposes for experiments in natural sciences: gathering knowledge, demonstration of phenomena, giving 'primary experiences' to pupils or the verification of a relation or model (cp. Kircher, Häußler &Girwidz, 2009). All these purposes lead to a better understanding of nature. Typically, there are up to six steps for an experiment in natural sciences. At first the object of investigation must be clarified. Afterwards pupils have to collect hypotheses as a second step. The third and fourth steps are the planning and execution of the experiment. Whilst execution the measurement of data is important in order to analyze these data for correlation between quantities. This analysis is the fifth step and is only followed by the last step: the interpretation of results. In the last step the results and hypotheses are compared (cp. loc. cit.). The interpretation of results itself often leads to another object of investigation and thus to another experiment. Even though there are differentiations between experiments (e. g., will pupils or the teacher execute? or in what phase of the lesson is the experiment integrated?) in natural sciences every experiment is about real objects.

There are similarities and differences between mathematical experiments and experiments in natural sciences. In both subjects an experiment describes a way of gathering knowledge by observation of controlled action with 'objects' (cp. Ludwig & Oldenburg, 2007, p. 4). The process of experimenting in mathematics is largely identical with the process in natural sciences. Though it is not necessary to collect hypotheses before trial. Examining several examples or handling with material is a starting point for pupils to build hypotheses, so step two can be replaced after step three and four. As mathematical facts need to be proved the sixth step of interpretation suggests approaches to a formal proof or leads to a repetition of the experiment with slightly different conditions (cp. Philipp, 2013, Goy & Kleine, 2015). Finally mathematical experiments can be detached from real objects. Thus, experimenting in mathematics needs the pupil to know heuristics and teaches process-oriented competences.

Mathematical experiments as a process

As mentioned before, experimenting in mathematics is a cycle of different steps. Referring to Philipp (2012) or Goy and Kleine (2015) there are four main steps:

- Stating the mathematical problem/question
- Generation of hypotheses
- Planning, execution and analysis of the experiment (short: 'trial')
- Elaboration of a mathematical model, concept or proof

For every experiment stating the mathematical problem or question is the first and the elaboration of a model, concept or proof is the last step. The order of the other steps can be changed considering the experiment's aim. If the experiment is about to verify or falsify hypotheses, those hypotheses have to be generated first. If the experiment aims on pupils learning how to experiment or making up their own models and concepts, the trial has to be placed before the generation of hypotheses (cp. Goy & Kleine, 2015, p. 5f).

Heintz constructs three contexts for mathematical experiments: discovery, validation and persuasion (cp. Philipp, 2013, p. 25). Context of discovery pertains to the generation of hypotheses and is meant as systematic trial in order to explore unidentified relations. Here knowledge is obtained by induction. Otherwise, knowledge is obtained by deduction when a given hypothesis is validated by the mathematical experiment. In this case the experiment is set in the context of validation. Finally, if neither discovery nor validation is needed because a relation, concept or model is already confirmed there is another context for mathematical experiment: persuasion. In this case the experiment shall convince the pupils (cp. loc. cit.).

Based on the theoretical background, enactive learning at the EnLeMaH-project can be described as hand-based activities, which enables pupils to discover mathematical relations or prove mathematical connections. The different phases of an experiment can be a guideline for teachers on the basis of the designing principles, to arrange an enactive learning situation.

SUMMARY

In this paper, an understanding of enactive learning has been laid that starts from the historical roots of Bruner and also focuses on the biological aspects of an active learning. The understanding of enactive learning should be concretized by looking at mathematical experimentation and its conditions. The understanding of enactive learning will be concretised by looking at mathematical experimentation and its conditions. Mathematical experiments find different approaches, this contribution was about the enactive approach. The explanations are intended to integrate the EnLeMaH project, to which this article refers. In this project, a training programme for teachers was developed to enable this enactive work synchronously or also asynchronously, even when learning at a distance. More information and access to the project can be found at www.enlemah.eu.

References

Artigue, M., & Blomhøj, M. (2013). Conceptualising inquiry-based education in mathematics. *ZDM–Mathematics Education, 45*(6), 797–810.

Brown, L. (2015). Researching as an enactivist mathematics education researcher. *ZDM–Mathematics Education, 47*, 185–196.

Bruner, J. S. (1966). *Toward a theory of instruction*. Belkapp Press.

Di Paolo, E. (2018). *Enactivismo*. Diccionario Interdisciplinar Austral. http://dia.austral.edu.ar/Enactivismo

Francis, K., Khan, S. & Davis, B. (2016). *Enactivism, Spatial Reasoning and Coding.* Springer.

Goy, A. & Kleine, M. (2015). Experimentieren – mathematische Zusammenhänge erforschen. *Praxis der Mathematik in der Schule*, (65), 2–8.

Kirchner, E., Häußler, P. & Girwidz, R. (2009). *Physikdidaktik: Theorie und Praxis*. Springer.

Maturana, H., & Varela, F. (1992). *The Tree of Knowledge: The biological roots of human understanding*. Shambhala.

Maturana, H. (1987). Everything is Said by an Observer. In W. I. Thompson (Ed.), *GAIA, A Way of Knowing: Political Implications of the New Biology* (pp. 65–82). Lindisfarne Press.

Lozano, M. D. (2014). La perspectiva enactivista en educación matemática: todo hacer es conocer. *Educación matemática*, *26*(1), 162–182.

Ludwig, M., & Oldenburg, R. (2007). Lernen durch Experimentieren. *mathematik lehren*, (141), 4–11.

Philipp, K. (2013): *Experimentelles Denken: Theoretische und empirische Konkretisierung einer Mathematischen Kompetenz*. Springer Spektrum.

Schunk, D. (2012). *Learning theories. An educational perspective*. Pearson.

Varela, F., Thompson, E., & Rosch, E. (1991). *The embodied mind: cognitive science and human experience*. MIT Press.