CAM CARPETS AS OUTDOOR STEM EDUCATION ACTIVITY

Xenia-Rosemarie Reit
University of Duisburg-Essen, Germany

Abstract. *Concepts can be better understood if teaching is combined with outdoor activities. Cam Carpets offer a unique real-life application of topics in physics and mathematics and can be realised by the students themselves. A Cam Carpets is an advertising carpet, that looks from a specific perspective like an upstanding advertising banner. The idea behind this effect is based on central projection. Thus, Cam Carpets are ideally suited as real-live application for geometrical optics in lower secondary physics education or of geometric analysis in higher secondary mathematics education. Especially instruction in analytic geometry is often very schema-oriented and lacks real application. The paper discusses the potential of a Cam Carpet project from an interdisciplinary and inter-year perspective. Case studies in physics and mathematics education and its project organisational aspects are reported. In addition, possibilities for internal differentiation for different levels of performance are pointed out.*

Key words: outdoor education, STEM education, digital technology, analytic geometry

INTRODUCTION

Cam Carpets are advertising carpets that look from a very specific camera position as if they were upright advertising banners. They are known from sports broadcasts, in particular football matches, or crosswalks. Cam Carpets in football matches are usually placed next to the football goals suggesting a three-dimensional advertising object to the audience at home (Figure 1). If not viewed from the right perspective, it is only a flat lying carpet on the floor with a skewed logo (Figure 1 right). The impression of an upright banner only appears from a single perspective, which is in Figure 1 the grand stand.

The principle of Cam Carpets is based on a (central) projection, in which spatial objects are displayed in two dimensions based on an initial position which is a fixed viewpoint. When advertising with Cam Carpets, this principle is applied to the opposite. A two-dimensional Cam Carpet image (Figure 1 right) creates a three-dimensional (mental) image for the observer (the camera). However, this three-dimensional image can only be seen from a specific viewpoint. From another position is seems to be neither upright nor legible. The three-dimensional letter in Figure 2 is not physically present but due to the representation it is mentally created by our eyes located in the camera position.

Figure 1: Left: Recording of Cam Carpets in the Commerzbank Arena Frankfurt viewd from the grandstand. Right: Cam Carpet identifyable as real carpet. (©Eintracht Frankfurt)

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Cam Carpets on the one hand offer an authentic application of mathematics and physics learned in school and on the other hand, they combine those contents with a result that is created by the students themselves outdoors. The latte is not mandatory as Cam Carpets can also be realised indoor but the aspect of creating a large-format Cam Carpet outdoors on the schoolyard which can be admired by the whole school community is an additional motivating factor. Another important feature of realising Cam Carpets outdoors is the implementation of mathematics in the real environment. Everybody knows how to construct a right angle with a triangular or how to draw dots in a coordinate system on quad paper. In the real environment a triangular will not help and the floor is usually not checkered. Authentic, real-world contexts drive the students' questions (Beames & Brown, 2016). Applying mathematics outdoors help students to understand mathematical concepts (Moss, 2009).

A deeper analysis of the Cam Carpet situation leads to geometrical optics or analytic geometry. Especially analytic geometry is a topic in higher secondary mathematics education which is taught mainly schema-driven (Filler, 2007; Borneleit, Danckwerts, Henn, & Weigand, 2001). Cam Carpets can be realised in small- or large-format with help of analytic geometry or geometric optics. Thus, they offer an impressive real-live application of mathematical and physical content. The Cam Carpet project can be realised with or without knowledge in analytic geometry. In the following, the modelling project Cam Carpets is described from a professional and teaching perspective from an inter-year and interdisciplinary perspective.

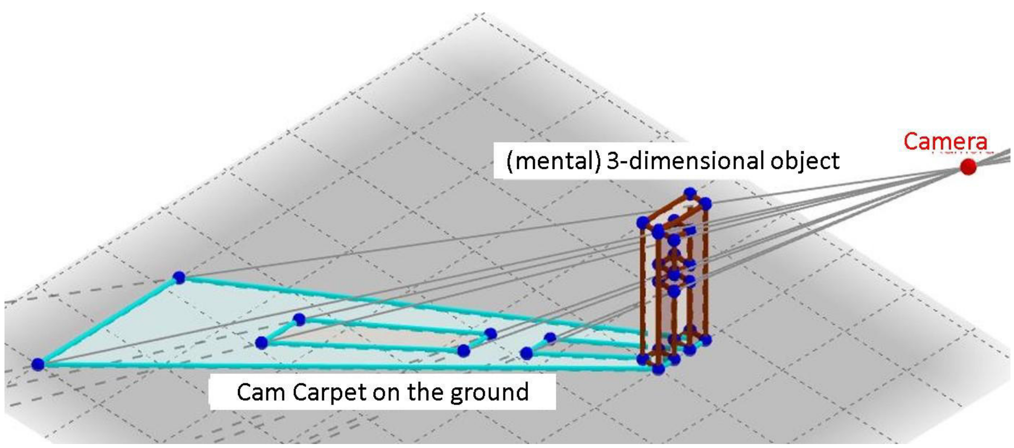

Figure 2: Central projection of a 3-dimensional letter originating from a fixed camera position.

CAM CARPETS IN LOWER SECONDARY PHYSICS EDUCATION

In lower secondary physics education light is approximated by rays. This is a model-based description which can explain related phenomena with sufficient accuracy. Besides conventional (shadow)experiments using common light sources like candles or torches, Cam Carpets offer a different and interesting approach to this light model. If considering the principle behind Cam Carpet as central projection, rays are represented as lines through a fixed point (camera point) conceivable as point-shaped light source (cf. Figure 2). Within the topic "shadows", Cam Carpets allow to penetrate those physical contents with an application used in reality. For high-performing students, considerations of linear functions in three dimensions can be a mathematical challenge and form a link to analytic geometry. The Cam Carpet project thus provides approaches for different performance levels and grades.

Implementation in class

As introduction a qualitative view of the principle of Cam Carpets via LEGO bricks and their shadows is to be considered (Figure 3). Shadows can be traced and attempts can be made to recognise the three-dimensional original LEGO figure by positioning the eyes at the light source. Limits of shadowing become apparent very quickly, which Cam Carpets can overcome: with shadowing only edges bordering translucent surfaces (cf. Figure 3). Other edges, for the three-dimensional impression necessary, can only be added retrospectively but not projected directly. Cam Carpets enable more detailed projections and better three-dimensional impressions.

The goal of a Cam Carpet illustration on the schoolyard should be clear to every group of students. In this phase the following questions must be asked. Which logo should be projected? Which dimensions should the final Cam Carpet have? How to find the projection points? Several possibilities are available according to the students' individual level of performance: GeoGebra (or other DGS), mathematically (taking up linear functions, for high-performing students) or through a handmade model using strings representing the projection lines? Which camera position is reasonable (considering the intended final dimensions)?

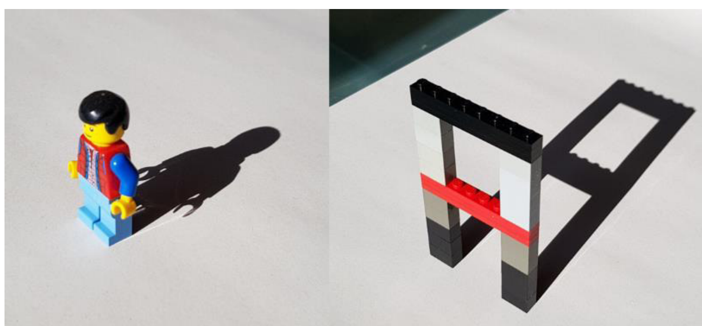

Figure 3: Introducing the Cam Carpet principle via shadowing (photo: D. Sommerbrodt).

Small format (A3/4) realisations can be created with little effort. A (smartphone)camera, tripods, GeoGebra (if applicable) to construct the projection points and a logo idea is needed. If the decision is to use a DGS like GeoGebra, a three-dimensional model has to be created in GeoGebra (Figure 2, left). Subsequently, the projection points can be generated by using the tool "lines" and "intersect" to create lines from camera point to every 3D-logo point and intersect those lines with the xy-plane (Figure 2). The simultaneous display of 3D-model and 2D-Cam Carpet (Figure 4) provides a lucid illustration. If positioned correctly the final Cam Carpet can be observed very impressively (Figure 5).

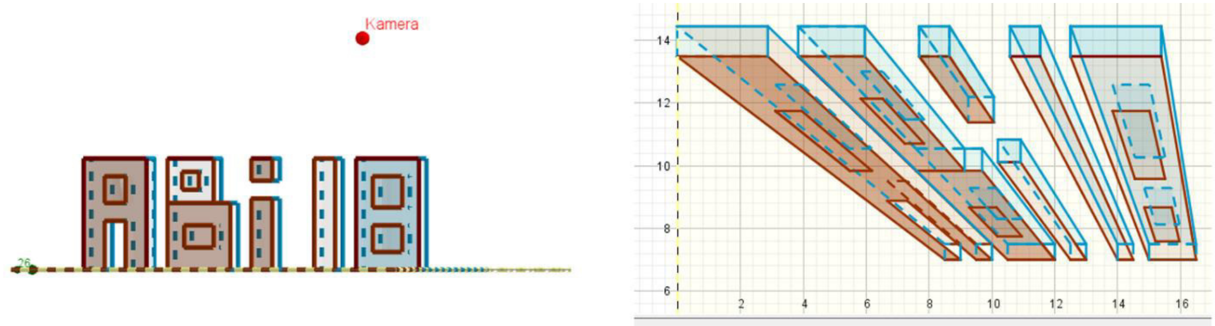

Figure 4: Simultaneous display of 3D-logo model and associated 2D-Cam Carpet.

How the Cam Carpets find their way to the schoolyard is described within the following chapter using a large-format Abi18 Cam Carpet.

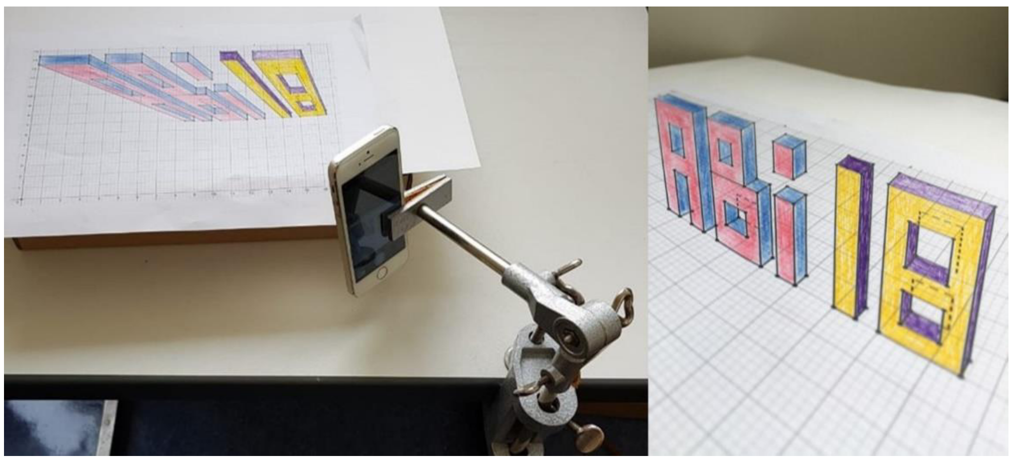

Figure 5: Left: Arrangement of Cam Carpet and tripod (ensuring a correct camera position) Right: 3D-impression of the Cam Carpet viewed from a fixed camera position.

CAM CARPETS IN HIGHER SECONDARY MATHEMATICS EDUCATION

Cam Carpets in higher secondary education provide the possibility for a modelling project worked out from scratch by students themselves with an impressive product result. The students decide on the logo to be developed within their project. Shortly before graduation (in Germany called Abitur), a lettering that can be used for the “Abitur”-newspaper is obvious and an additional motivation factor that makes the modelling project the students' own. When implementing the modelling project, the teacher assumes the role of a moderator, who keeps an eye on the individual project parts and their merging in terms of organization and time.

The aim of the modelling project for the higher secondary class was to create a large format Cam Carpet logo on the schoolyard (Figure 6). The 22 students of the mathematics course, who are about to graduate, opt for "Abi18". A total of four 90-minute courses and the homework time are available to develop and realise the Cam Carpet. A lot of preparation has to be done before the Carpet can be drawn on the schoolyard. A possible Cam Carpet position on the schoolyard, including a camera position, must be found in order to then determine the coordinates of the letter points projected on the xy-plane. Those coordinates depend on the camera position. Following those organisational aspects, the student groups started to create a 3D-model with GeoGebra3D.

The mathematics behind the Cam Carpets can be located in analytical geometry. Whether with track points, projection vectors or, usually at the level of the advanced course, with projection matrices – Cam Carpets allow performance-differentiated work. The following alternatives are described as examples for a point that is to be projected from a camera position onto the xy-plane (Figure 2).

Figure 6: Large-scale "Abi18" Cam Carpet on the playground (letter height ca. 7m).

Approach 1: Track points

The camera is in $K(5|-5|6)$ and a point of the 3D model is $A(0|0|4)$ (Figure 2). From this, a line g_{AK} can be determined that contains both points. $\overrightarrow{OA}$ forms the support vector and the vector $\overrightarrow{AK}$ represents the direction vector of the line g_{AK}.

(Eq 1) $$g_{AK}: \vec{x} = \overrightarrow{OA} + r \cdot \overrightarrow{AK} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} + r \cdot \begin{pmatrix} 5 \\ -5 \\ 2 \end{pmatrix}, r \in \mathbb{R}$$

The projection point of A in the xy-plane is obtained by determining the track point S_{xy} of the line g_{AK}. The track point has the z coordinate $z = 0$. The z-coordinate of the general line point is $z = 4 + 2r$. In the xy-plane, $0 = 4 + 2r$. This gives $r = -2$. The track point S_{xy} can now be obtained by inserting $r = -2$ into the line equation:

(Eq 2) $$\vec{x}_{xy} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} + (-2) \cdot \begin{pmatrix} 5 \\ -5 \\ 2 \end{pmatrix} = \begin{pmatrix} -10 \\ 10 \\ 0 \end{pmatrix}$$

This leads to the track point $A_{xy}(-10|10|0)$, which is also the projection point of the original point A. Analog calculations with all points of the 3D logo lead to the coordinates of the 3D model projected onto the xy-plane (cf. Figure 2).

Approach 2: Camera point dependent projection vector

Since many projection points have to be calculated, the pupils quickly discovered the general pattern behind approach 1 and developed an efficient method from it: After deriving a general projection vector that depends on the camera position, it can be used for the calculation of the projection points by inserting the points to be projected and the camera point. The implementation of this projection vector in a spreadsheet program, similar to the way implemented by a group of students with variant 3, enables an efficient determination of the projection points.

In general, a letter point $B(x|y|z)$ with a camera at point $K(k_1|k_2|k_3)$ in camera direction $\vec{v} = \begin{pmatrix} k_1 - x \\ k_2 - y \\ k_3 - z \end{pmatrix}$ on the xy-plane are projected, which creates the point B_{xy}. The approach results from the general intersection problem (see approach 1)

(Eq 3) $$\vec{b} + r \cdot \vec{v} = \overrightarrow{b_{xy}}, r \in \mathbb{R}$$

and thus,

(Eq 4) $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} + r \cdot \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} x_{xy} \\ y_{xy} \\ 0 \end{pmatrix} = \overrightarrow{b_{xy}}$$

The third coordinate of $\overrightarrow{b_{xy}}$, solved for r, returns

(Eq 5) $$r = -\frac{z}{v_3}$$

Insertion of Eq 5 in Eq 4 yields

(Eq 6) $$\overrightarrow{b_{xy}} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \left(-\frac{z}{v_3}\right) \cdot \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} -z \cdot \frac{v_1}{v_3} \\ -z \cdot \frac{v_2}{v_3} \\ -z \end{pmatrix} = \begin{pmatrix} x - z \cdot \frac{v_1}{v_3} \\ y - z \cdot \frac{v_2}{v_3} \\ 0 \end{pmatrix} = \begin{pmatrix} x - z \cdot \frac{k_1 - x}{k_3 - z} \\ y - z \cdot \frac{k_2 - y}{k_3 - z} \\ 0 \end{pmatrix}$$

The track point searched has the general coordinates $(x - z \cdot \frac{k_1 - x}{k_3 - z} | y - z \cdot \frac{k_2 - y}{k_3 - z} | 0)$, where $(x|y|z)$ is the coordinate of the point to be projected and $(k_1|k_2|k_3)$ represents the coordinates of the camera point.

If here the point $A(0|0|4)$ to be projected and the camera point $K(5|-5|6)$ from approach 1 are used, the projection point A_{xy} of A is obtained

(Eq 7) $$\overrightarrow{a_{xy}} = \begin{pmatrix} 0 - 4 \cdot \frac{5}{2} \\ 0 - 4 \cdot \frac{-5}{2} \\ 0 \end{pmatrix} = \begin{pmatrix} -10 \\ 10 \\ 0 \end{pmatrix}$$

and thus, the same result as with approach 1 (Eq 2).

Approach 3: Camera dependent projection matrix

On a higher level, a camera point dependent projection matrix can be derived from Eq 6 for calculating the projection points. This projection matrix, equal to approach 2, can be used for the calculation of the track points (projection points). Using the matrix notation, Eq 6 is rewritten as follows

(Eq 8) $$\overrightarrow{b_{xy}} = \begin{pmatrix} x - z \cdot \frac{k_1 - x}{k_3 - z} \\ y - z \cdot \frac{k_2 - y}{k_3 - z} \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & -\frac{k_1 - x}{k_3 - z} \\ 0 & 1 & -\frac{k_2 - y}{k_3 - z} \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = P \cdot \vec{b}$$

where P is the projection matrix sought, which maps an arbitrary point B depending on the camera position on the xy-plane.

For the initially considered point $A(0|0|4)$ and the camera position in $K(5|-5|6)$, the following projection matrix results, which is dependent on the camera position

(Eq 9) $$P_A = \begin{pmatrix} 1 & 0 & -\frac{5}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 0 \end{pmatrix}$$

Inserting in Eq 8 results

(Eq 10) $$\overrightarrow{a_{xy}} = P_A \cdot \vec{a} = \begin{pmatrix} 1 & 0 & -\frac{5}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} -10 \\ 10 \\ 0 \end{pmatrix}$$

thus $A_{xy}(-10|10|0)$ which is the same track point as with the projection vector in Eq 7 and previously calculated with approach 1.

Since the projection vector and the projection matrix both depend on the camera point, a calculation must be made for each projection point using these approaches. In order to make the work easier, a group of students used a spreadsheet program in variant 3, in which the projection matrix was implemented to automatically calculate projection points.

Implementation in class

For a large-scale implementation of a Cam Carpet on the schoolyard four 90-minutes courses and a full school day was necessary. A large-scale implementation on the schoolyard requires preliminary organizational considerations, but rewards with an impressive result for the whole school community.

It is important to take into account the space available on the school premises. The larger the individual letters of the logo, the more impressive the result, but large letters also mean more drawing and material. The projection letters of the Cam Carpet in Figure 6 are $3m\ x\ 7m$ in size, with the camera positioned at a height of $5.6m$. The camera position from which the Cam Carpet can be viewed should be chosen so that it is accessible to a wide audience in order to give the whole school community access to the artwork.

Once all the letter coordinates have been determined, they can be realized on the schoolyard. Before the coordinates can be drawn in, a coordinate system must be placed on the schoolyard. The perpendicularity of the x- and y-axis of a coordinate system on a "non-checkered" surface that is sufficiently large for the Cam Carpet is a real challenge. After some consideration, the students decide to use a twelve-knotted cord (3-4-5 triangle) and apply their knowledge of the Pythagorean theorem (or its inversion) and Pythagorean triples from lower secondary mathematics: If you stretch a cord with the edge lengths

$3:4:5$, there is a right angle between the two shorter edges. This allows a right angle to be constructed with sufficient accuracy.

Another problem is the sufficiently precise drawing of the letter coordinates on the 18mx15m area. The students need a check pattern that on the one hand allow the coordinates to be drawn in with sufficient accuracy, but on the other hand is not visible in the later image so as not to disturb the 3D impression. The use of a chalk line (marking line) creates the desired and at the same time only slightly visible check pattern. Available in a conventional hardware store, a chalk line consists of a housing filled with colored chalk in which the line is wound up. Creating $1m\ x\ 1m$ checks is a sufficiently precise scaling.

A good contrast between the front and side surfaces of the letters is important for the 3D effect in order to be able to clearly differentiate between the respective surfaces. To estimate the amount of street chalk to be bought, the students used GeoGebra to have the area of the 2D projection displayed. A test with a common street painting circle cylinder shows that this is sufficient for an area of approximately $1.5\ m^2$.

After having finished the work, the 3D impression cannot be seen from the schoolyard. The surprise is great when looking at the Cam Carpet from the camera position. Many photos are taken with students sitting "in" the letters or "leaning on" them. But the most impressive is the jump video/photo from Figure 6.

SUMMARY AND OUTLOOK

The modelling project "Abi18" Cam Carpet allows not only performance-differentiated, but also inter-year and interdisciplinary project work due to the variety of solution and implementation options. The implementation in lower secondary physics education produces impressive results without mathematical penetration of the underlying principle. As part of analytical geometry, Cam Carpets offer a motivating application for the otherwise all too often schema-driven subject area and additionally reward with an impressive result for the whole school community if implemented in large format on the school yard.

Human vision and the effect of optical illusions allow an exciting expansion of the topic in the direction of biology, physiology and psychology. Further considerations regarding the quality of the 3D impression depending on the deviation from the calculated camera point as well as the effect of binocular vision as the basis of 3D vision give reasons for an interdisciplinary expansion of the subject.

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