MAN IS THE MEASURE OF ALL THINGS - MATH TRAILS IN LYON

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Abstract. *We report on our 3 years long experiments in Lyon of Math trails training sessions in the realm of the Erasmus+ MoMaTrE project. We trained students to open a scientific eye on the world around them and envision their own body as the ultimate tool to make sense of it. In particular we present here the results of calibration of body parts by students, their anthropometric identity card, a statistical analysis of the results, comparing it to the Vitruvius antique esthetical cannon, discussions among students and the different strategies proposed by groups of students.*

Key words: Outdoor mathematics, Modelling, Mathematical trails, Embodied Mathematics, Measure

INTRODUCTION TO MATH TRAILS AT UNIVERSITÉ CLAUDE BERNARD LYON 1

Why teacher training with Math Trails?

Opening a scientific eye on the world around us might be the primary benefit of a science faculty education. In this rapidly evolving world, short sighted technical applications don't last long, and educating students as scientists, helping them question the universe around them and finding answers to the questions they deem interesting enough to investigate is a real challenge.

Opening a scientific eye on the world is important for science students and all the more for pre-service and in-service mathematics teachers that have to recognize, in the concepts they teach students, not only scholastic material, but as well tools that were shaped in a certain context in order to solve problems.

Our Institute for Research on Mathematics Education (IREM) in Lyon is involved in training in-service and pre-service teachers as well as university students. We describe in this article lessons learnt during the last three years regarding teaching Mathematical Trails in Lyon, especially training students to measure with their own body. We insisted on that point in order to cause appreciation of measures uncertainty among students. The main result is a confirmation of the resistance of punctual conception of measures despite formal training on measurement interval. Vitruvius ideal human proportions is put to the test with regard to the collected data.

Who we are, what we do

The Institute for Research on Mathematics Education (IREM) in Lyon has been engaged in Realistic Mathematics Education for years, specializing in Long Open Problems (Aldon, 2018; Arsac & Mante, 2007), as a teaching tool in the classroom. In 2000, we celebrated the UNESCO year of mathematics with a large mathematical trail in the city and a mobile exhibition "Why Mathematics" that we are still touring in the region, visiting one school each month. Since 2006 we have been organizing a regional mathematical competition, where around 30 000 secondary students yearly engage in teams to solve challenging problems. We participate in the House of Mathematics and Computer Science (MMI), a peculiar place, mix of a museum of mathematics, an art gallery and a training center, that federates energies around the diffusion of mathematics in the region and we organize

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events where mathematical trails are featured in many occasions such as Math.en.Jeans congresses.

Within the Erasmus+ Mobile Math Trails for Europe (MoMaTrE) and now Math Trails in School, Curriculum and Education Environments of Europe (MaSCE[3]) we train students and teachers to create and run mathematical trails. The students we train are undergraduate science students, pre-service mathematics teachers and pre-service primary school teachers. We conducted as well in-service teacher trainings but they are not covered in this article.

Typical training course

During a typical training, students create groups that have to:

- run a math trail, composed by selecting previous years students productions, using the mobile app MathCityMap (often in *Digital Classroom* mode for motivation);
- be trained in mastering the authoring platform (mathcitymap.eu);
- get out in the field in order to come back with pictures, measurements and lots of ideas for designing tasks;
- enter their tasks in the platform, sharing them as a group and setting up a trail;
- run, evaluate and report on the math trails of their fellow students; pairings are done in a way to maximize interactions and feedbacks;
- design and conduct their own math tasks and trail in other locations;
- report on their experience and justify the choices they made, especially diverse ways of tackling the problem, the theoretical tools they used and the associated levels of students.

Designing realistic tasks for students is not easy (Fessakis et al., 2018; Siswono et al., 2018). When *running* a mathematical trail, students are initiated in asking themselves how they can use their knowledge to solve problems; when *designing* math trails, they are *problem posers* and question their environment. In both situations, modelling a phenomenon is the crucial point. Students have to identify the relevant quantities and information, choose what to neglect and what to focus on, name items, make assumptions regarding their relations, predict, approximate, simplify, estimate, sketch diagrams, graphs, tables to collect data, measure, make statistical inferences, compute, interpet, verify, revise... All that is at stake when running or designing a math trail is of great complexity but this broad view on modelling, while guiding our teaching, is not the focus of this article. We refer to the literature for more details (Buchholtz, 2017; Cahyono et al., 2020; Cahyono & Ludwig, 2019, 2018; Druken & Frazin, 2018; English et al., 2010; Fessakis et al., 2018; Gurjanow et al., 2019; Lehrer & Schauble, 2007; Richardson, 2004). Our main point is about relating a specific moment in the course, the introduction of the training, when we teach how and why our body can be used to estimate quantities, and what we learnt from the data we collected.

To briefly give an overview of students' production, trails are located mainly on the UCBL training campuses (La Doua or INSPÉ-Lyon or Saint-Étienne) and near the schools where students teach as trainees. We had 445 tasks designed on the main campus, 38 in INSPÉ-

Lyon, 24 in St Étienne, and 143 in different schools where pre-service teachers conduct their internship.

The location of tasks on the campus is interesting, showing the main locations where students tend to flock around, library, teaching halls, sports halls, parks… and very few pieces of interest are left unchecked after three years on the spot. In order not to submerge the area with visible public tasks, most tasks are kept private.

In a given designer group, we ask students to work together at different levels, finding tasks for their little sister, for their cousins, for their parents, envisioning a scenario, telling a story and addressing different educational levels. This requirement helps students realise what is taught at which age and what it is good for as a problem solving tool. This is somewhat reversed compared to Realistic Mathematics Education techniques where reality should come first: here we require to take into account notions as tools to tackle problems.

In our experience, most questions that students come up with are not realistic in the sense of Realistic Math Education (RME) (Freudenthal, 1968), very few tackle a compelling issue, and most students simply figure out fun questions to quizz their fellow students, but anyway, we believe that having fun with math is a good start to change one's view on what mathematics *is* and what it is to *do* mathematics, even though we have not tried to objectify this belief.

ANTHROPOMETRIC IDENTITY CARD

Before the brief presentation of the mobile app, the actual introduction of the course is done through an enigmatic Identity card. The students are equipped with some measuring devices such as folding rulers and measuring tapes, some string, and are asked to fill in a card that will serve them in the field.

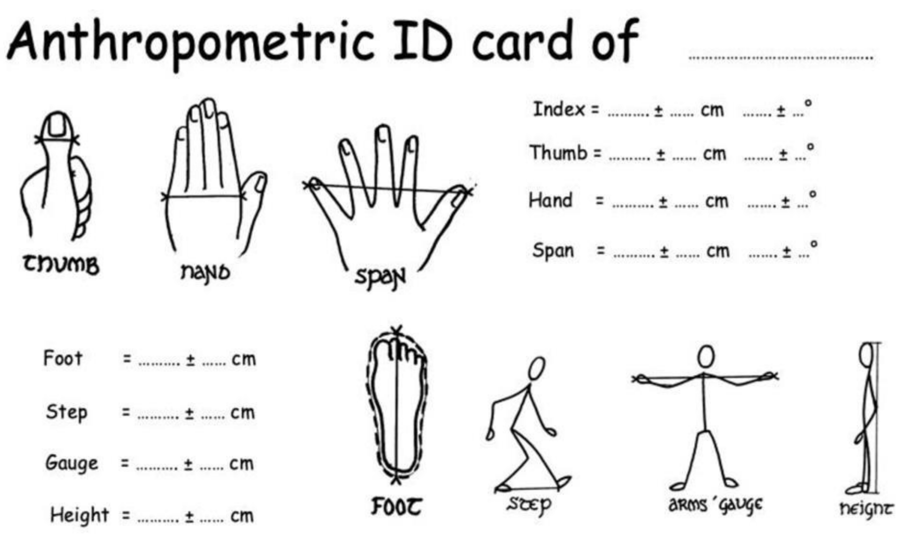


Figure 1: Anthropometric Identity Card.

In a nutshell, students are asked to calibrate their own body in order to perform measurements in the field. Some students are really reluctant at first, saying that such measures are going to be inaccurate, that a ruler gives you THE right measure. It requires some persuasion to make them evolve on the idea of what really is a physical quantity, a measure of it in a given unit and the measurement, the result of actually measuring it with a measuring tool. What follows is the recollection and unfolding of this process, where very complex yet profoundly naturalized ideas about quantities, their measure and incertitudes are discussed with students. We follow the chronological order of the course exposition.

Why do we measure things?

Man has always been counting and measuring, from dividing a stock to planning a trip to the moon, measuring serves to plan, to memorize and communicate about quantities, to validate equitable repartition and transactions, to predict what is likely to happen, in a word to better **understand** the world around us. Science, based on Aristotelian qualitative observation, is now first and foremost a science of measure and its scientificity is based on the fact that measures can be reproduced and verified (Chambris, 2008; Munier & Passelaigue, 2012).

What is a measure?

From the different representations and ideas of the students, we tend to converge after negotiations, towards the following definition, somehow compatible with the International Metrology Vocabulary:

*Measuring is finding the **ratio** between the magnitude of a quantity and the standard magnitude in a system of reference.*

Different cultures established different standards but the use of the human body as a gauge is prevalent, with units such as *foot*, attested since neolithic ages. Students' impression always is that these type of "standards" won't allow for accurate comparisons because it is not universal and not reproducible by another person. Indeed, one needs to calibrate one's own body in order to use it as an accurate tool to evaluate physical quantities, especially lengths. Therefore our first task was to have the students fill in an individual anthropometric card. And this card accounts for an evaluation of the accuracy, with an estimation of uncertainties, the first approach being to round at half a graduation on the ruler.

How to find a measure by direct comparison?

This anthropometric data serves as a calibration for a measurement protocol to be used on the field. For each protocol and each measuring instrument (part of the body) we can compare the result to the value given by a more standard protocol using a ruler, itself vitiated by errors. These errors are of two major kinds: systematic errors (the measure we made, how accurate it could be, is wrong because our protocol or our instrument are faulty) and precision errors (because the right protocol is not well applied and there are some random variations). In a short amount of time, without weather or wear and tear variations, we should consider our instrument to be stable and faithful by looking at the statistics of the results and smoothing out the random dispersion.

This gives a new insight on the choice of body parts: one should be able to observe it (so one can not use one's own nose for example) and it should lead to a robust protocol with a reduced randomness. It should be stable, easily comparable and usually based on an extremal point, where positioning error is of second order. Of course, adequacy is paramount, and the anthropometric card is usable for trails in the city or on the campus, much less in the classroom where a ruler is to be used.

By reproducing the same protocol with the same instrument, students get almost the same result, it is reproducible, but not the same as with another instrument, that is another student. Reducing their own precision errors, students tend to overlook systematic errors and abate the uncertainty of their measures. This point will be seen again during the conception of math trails with the introduction of intervals of validity.

Measuring Errors

At this point of the course students should understand that whatever measure they end up with is not THE correct actual value but they have to choose a bracket of uncertainties for accepting correct answers. But this adoption is very shallow, they still mostly tend to propose narrow options, even when fellow students consistently tell them that they measured something else. As pointed out by (Volkwyn et al., 2008) among others, the *punctual* reasoning, accessible with only one measure prevails and resists over the *statistical* approach. End of course satisfaction questionnaires often report that students confronted to the dismay of their fellows, unable to fall in the expected "good" interval, provided a firsthand experience of their own naive realism. Whether it does evolve towards scientific realism, in a more concrete way than a lecture on epistemology and the Nature Of Science is yet to be confirmed as (Otte et al., 2019) points out, despite teacher's conviction, hard evidence might be deceiving.

Evaluating a bracket

Measuring a length with a ruler for example, it is clear that the accuracy can not exceed half a subdivision, hence all the values computed from it have to take this error into consideration. On simple additions these positive errors never subtract but add up, but are often multiplicative yelding the so called "errors propagation". In more complex formulae, chain rule of derivation is useful. See (Buffler et al., 2008)

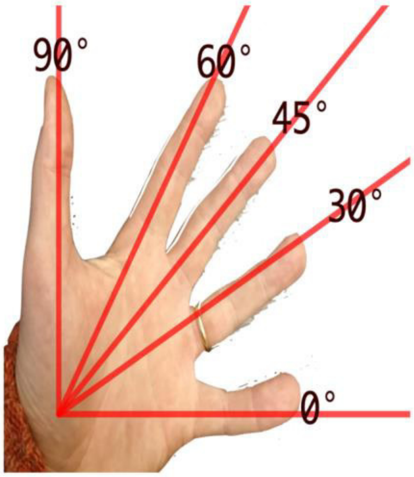
Ten Meters are not Ten Thousand Millimeters

Given the misplaced certainty of their accuracy on the part of students, we setup challenges to measure a given physical quantity, such as the length of a corridor, using different tools, whether or not adequate, and experiment the spread of the actual values. Doing so, we validate the fact that steps are as accurate as a folding ruler when measuring lengths of around a decameter. For different orders of magnitudes, different instruments are adequate.

Students are hence set to calibrate their body as a universal measuring tool. Each self-constituted team is invited to come up with their own strategy. To begin with, their first reaction is to measure what is indicated on the ID card: the width of just ONE thumb, the length of ONE step instead of calibrating it as it will be used in the field, that is to say measuring a sufficient quantity of it in order to gain precision by reducing dispersion of each individual unit. We continue to witness the resistance of the punctual conception of a measurement, if not on a single measure, then on the dispersion of measures.

Angles

Whereas estimating lengths is within every student culture, the issue of estimating angles usually opens up gaps in students comprehensions. What can be the angle associated with the span of your hand?



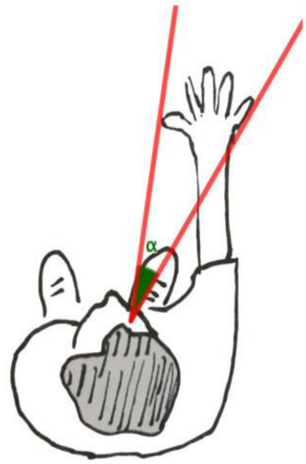


Figure 2: Rough angles. Figure 3: The span angle.

Some have heard about angles made by fingers in an open hand, which are very crude estimates (see Fig. 2). It is not at all what we are asking here. Uneasiness and awkwardness in front of the vague and obscure task, grow in the class, which is typical of the usual didactical contract of math classes where a univocal (albeit not that clear) question is asked, only by the teacher, who already knows the only possible answer to it, and the task is to find how to apply the knowledge at hand in order to please the teacher and get a good mark. Stating that being a scientist is to be able to navigate in the unknown and staying the course is not enough to alleviate students embarrassment in front of the difficulty to model the situation, to give definitions to the terms. Other such embarrassments are yet to come when students will face their first tasks assignments: "How on earth am I supposed to know that?! I have no idea!" And insisting at this point of the course does ease subsequent engagement in the field.

In order to convey what we have in mind, and abate the confusion a little, we usually show the class that, closing one eye and extending your arm, sitting in a corner of a room, you can fit pretty accurately five extended palms from one corner of the room to the next, so that the span of your palm, at arm length, seen from your eye, is approximately 18°. Each one tries and can sees for herself what it means. What we mean is the angle between two vertical planes, meeting in your eye and tangent to whether the left or the right end of the span of your hand.

But to go from this crude estimate to the measure of the angle of your thumb or your index, other strategies have to be devised, you cannot possibly add up around eighty indices in order to make a square angle?! Laying down a drawing is usually the key to understanding.

We are talking here about physical quantities, but making a sketch is usually a good creativity unblocker even in abstract frameworks.

The sketch of the light rays, the position of their intersection point, reveal widespread naive views on vision, leading to discussions on the size of an eyeball and other (yet to be proved irrelevant) topics.

Teasing students using social media, especially those focussing on pictures, such as Instagram, the notion of *Horizontal Field of View* (HFoV) of an image does eventually pop up in the debates. Trying to estimate roughly the 114° of horizontal monocular view of the human eye is a fun exercise. Having students picture what it means usually leads to the right description with the central position of the closed eye and the rays leading to it. But even in this framework, realizing what that implies for the angle of objects *blocking* the view is still another matter.

Trigonometry For The Win

At this point, asking questions such as: *what is the angle made by the door seen from the end of the classroom* does find correct answers. But the *opened* field of view doesn't seem to be as clear for a *blocking* field of view and it requires a lot of conceptual efforts for many students to be able to figure out what it means to measure the angle blocked by an index, a thumb or the span of a hand: it means to measure the angle of the object that is exactly *covered* by this part of the body at outstretched arm length.

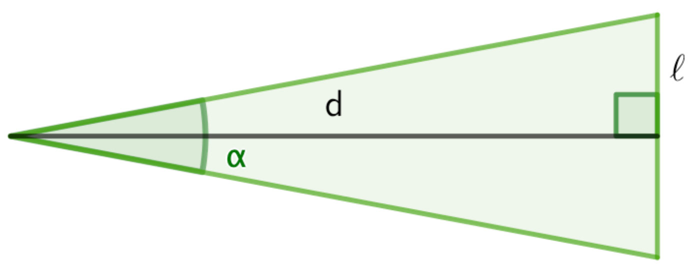


Figure 4: A thin triangle.

In Fig. 4, evaluating the field of view α is based on evaluating the size 2ℓ of an object, which is hidden by (say) our thumb at a distance d, subject to the formula $\tan \alpha/2= \ell/d$. But for small angles, there is no need for a sophisticated calculator because the measure α in radian is simply the ratio of lengths $2\ell/d$, and it is equal to its tangent and to its sinus up to negligible differences: $\tan(\alpha)=\alpha+o(\alpha^2)$, a formula that all students know but that seems to be largely out of reach in this situation. Hence, whereas the square angle on projection is of a crucial importance, its position right in the middle of the object is not relevant and $\alpha= 2\ell/d \pm 2(\Delta\ell/d+\ell\Delta d/d^2)$ in radian. These very simple observations cause tremendous discussions.

STATISTICAL ANALYSIS

The Vitruvian Man (see Fig. 5) is a drawing by Leonardo da Vinci, representing ideal human body proportions in the Renaissance, based on the work of Vitruvius (Ist century BC), a Roman architect. The described proportions are: the arm's gauge is equal to the height giving the equal sides of the square, from the tip of the finger to the elbow is ⅜ of the height, the foot is 1/7th of the height. Face and shoulder proportions gives a distance from the eye to the hand of about 42% of the height. Since we collected the anonymized anthropological data of the students, we can put these ideal Renaissance figures to the test. We collected 218 entries, evaluating 12 measures: the widths of the index, the thumb, the hand palm, the hand span, in cm, then their associated angles at outstretched arm length, the foot, the step, the arm's gauge and the height. We are in position to question whether these proportions are cultural or still relate to XXIst century young adults.

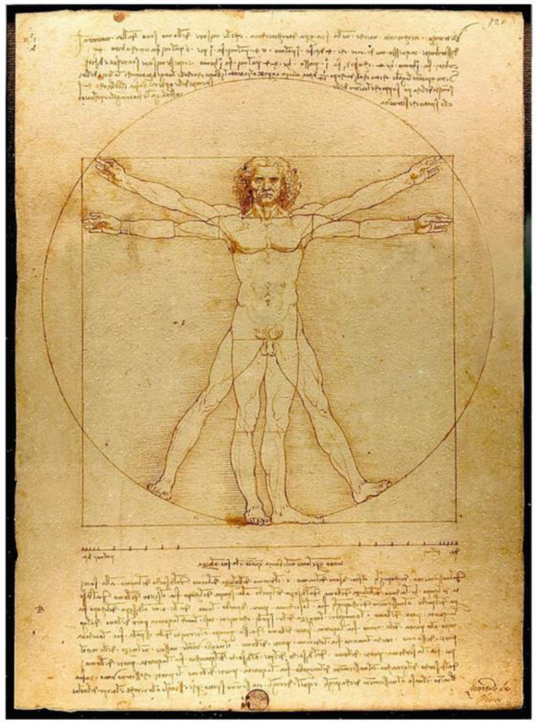


Figure 5: The Vitruvian Man by Leonardo da Vinci, c. 1490.

Another use of statistics is to infer some strategies taken by students. For example, we can compute the distance between the eye and the hand given the ratio of the size of the thumb and the associated angle, likewise for the index, the palm and the span. The deviation between these figures is informative, it is sometimes a constant, meaning that students didn't go into lengthy evaluations of angles in the way we tried to convince them but simply by estimating this arm length and computing (not measuring) the angles given the width of their thumb/index/palm/span. Sixteen students have a standard deviation less than 5% between these four measures and among them four have a zero deviation and many others having very high deviations due to bad protocols (see Fig. 6).

A remarkable result of the analysis is that the distributions of some angles are quite narrow, whether short or tall, most students have approximately 1.6° as an index field of view, 2° for their thumb and 7.2° for their palm. The dispersion of the hand span is much wider (see Fig. 8-9). That means for example that your hand, when stretching your arm and aligning its bottom part with the horizon, corresponds roughly to a half hour of sun's course in the sky, because it takes about 12h from sunrise to sunset, corresponding to 180°, that is 15°/h. More precisely, your index amounts to about 6 minutes, and two thumbs for a quarter of an hour.

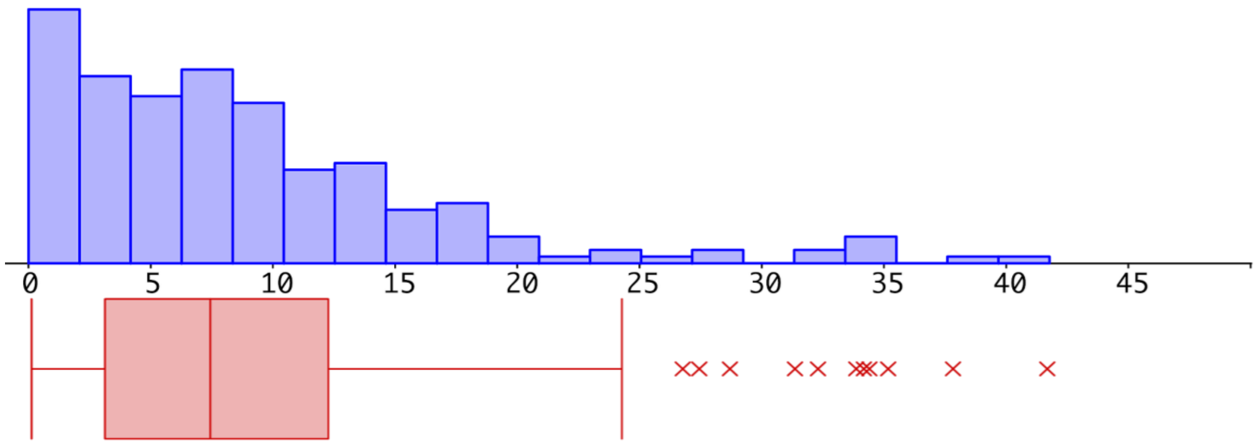


Figure 6: Standard deviation of eye/hand distance (cm).

Of course, these estimates vary with the season, days are longer in summer, shorter in winter and with your latitude (variation between 9h to 15h of daylight in France).

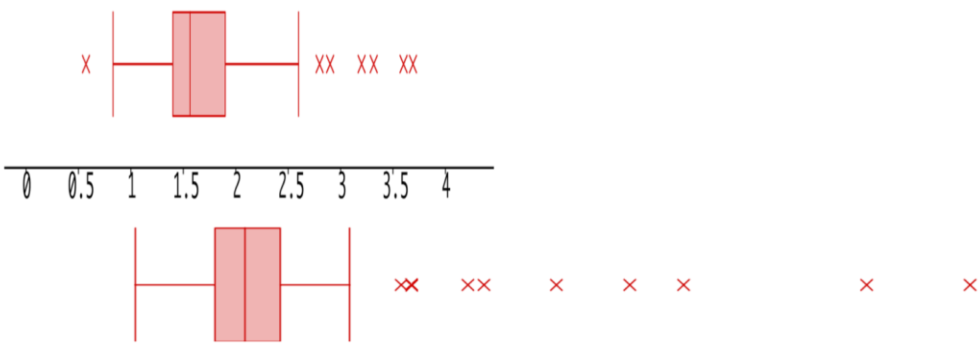


Figure 7: Box plot of index and thumb angles (°).

Dividing all lengths by the height, one can compare the dimensionless data to Vitruvius standard proportions. Some students were already aware of the correspondence between arm's gauge and height and indeed there are 40 cases of exactly same values in both fields. But even without these data, the correspondence is really very good, with a mean and median values of 1 and a standard deviation of only 4%. The foot correspondence is as well very good in terms of dispersion, with less than 10% standard deviation, but since students weren't bare foot, Vitruvius proportion is lower, students' foot, *with their shoes on,* is in a proportion of 7.8 with their height, not 7. The arm's length inferred from the fields of view appears to be more diverse than the one proposed by Vitruvius, with a much higher dispersion, due as well to many bad students protocols (see Fig. 9).

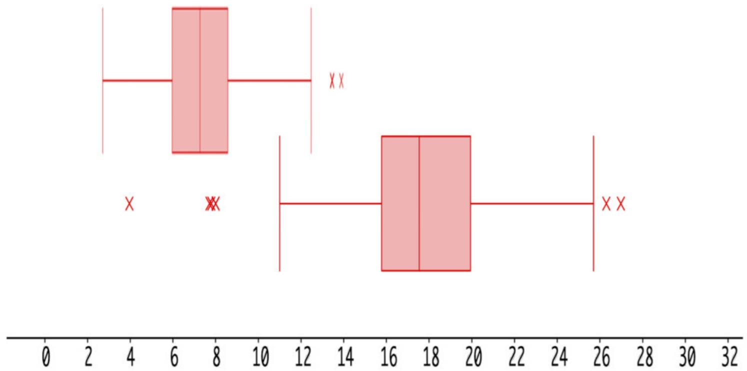


Figure 8: Box plot of the angles, of palm and span of the hand (°).

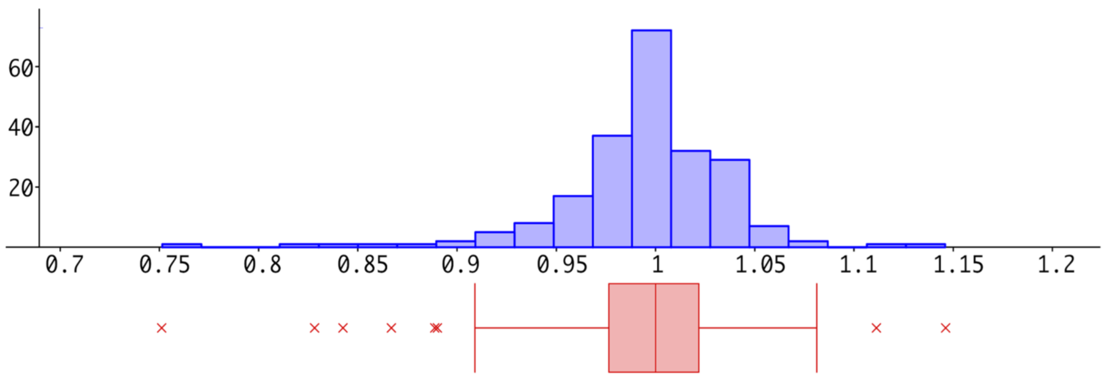


Figure 9: Dispersion of ratio arm's gauge/height

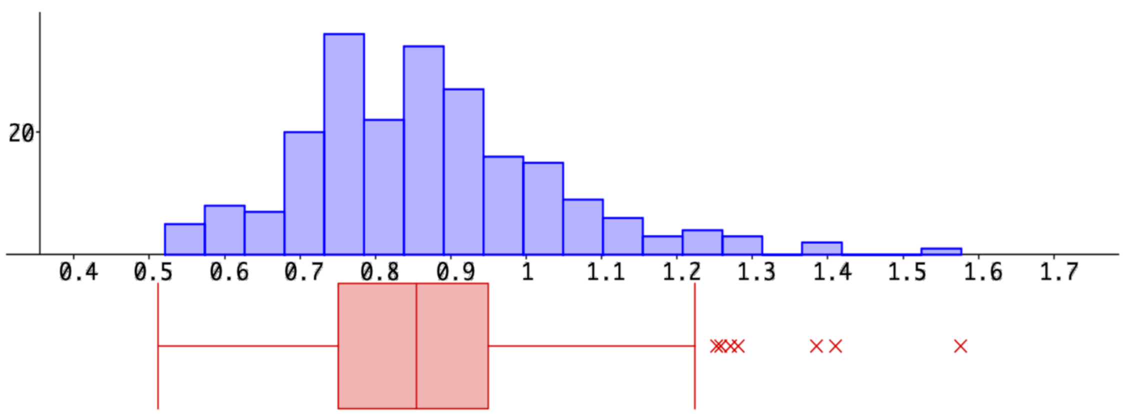


Figure 10: Dispersion of ratio of the distance eye/hand with Vitrivius proportion

CONCLUSION

In this article we have seen that math trails and specifically anthropometry can be a good incentive to open a scientific eye on the world around us, to engage students in a reflexion on exactness of measurements and scientific reasoning. Moreover, we can see that Vitruvius is still right: XXIst century human body continues to follow specific proportions.

Additional information

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