MATH TRAILS THROUGH DIGITAL TECHNOLOGY: AN EXPERIENCE WITH PRE-SERVICE TEACHERS

Ana Barbosa and Isabel Vale Instituto Politécnico de Viana do Castelo & CIEC/Universidade do Minho, Portugal

Abstract. This paper describes part of an on-going study that aims to understand the potential of digital technology in outdoors mathematics from the perspective of pre-service teachers. We followed a qualitative approach and data was collected through observation, two questionnaires and photographic records. The study involved forty-eight participants that used Math City Map to do a math trail in the city centre of Viana do Castelo. Results show that they valued the experience, highlighting the possibility of solving realistic problems, developing cooperative work, critical thinking and the establishment of mathematical connections. They found the app to be user friendly and motivating, mentioning its contribution for students' engagement through active learning, spatial orientation, autonomy and being more interactive then the paper version.

Key words: Math trails; Problem solving; Mathematical connections; STEM education; Teacher training.

INTRODUCTION

This paper has a strong support on previous work developed by the authors in the scope of outdoor mathematics. Several studies conducted with pre-service teachers (e.g. Barbosa & Vale, 2016; Barbosa & Vale, 2018; Vale, Barbosa & Cabrita, 2019) have shown that the outdoors can be seen as a privileged educational context, which promotes positive attitudes and additional motivation for the study of mathematics. In particular, math trails have great potential for making more visible the connections between mathematics and everyday life, specifically the environment that surrounds us. These studies focused mainly on a particular detail of the math trails and that was task design, approaching different aspects of problem posing, using a mathematical eye to formulate tasks that highlight connections with daily life. Being part of the Consortium of the Project Math Trails in School, Curriculum and Educational Environments in Europe (MaSCE³ - part of the Erasmus+ Programme, Key Action 2 - Strategic Partnerships under the number: 2019-1-DE03-KA201-060118), gave us the opportunity to contact with a different approach to math trails, adding the possibility to resort to digital technology. The use of Math City Map (MCM), a project of the working group MATIS I (IDMI, Goethe- Universität Frankfurt) in cooperation with Stiftung Rechnen, has been reported as having a positive impact in supporting teachers and students in the process of teaching and learning mathematics outside the classroom (e.g. Cahyono & Ludwig, 2019; Ludwig & Jablonski, 2019). Having the conviction that these approaches are extremely relevant in mathematical education and also in the development of certain skills expected from students in the 21st century, it is our purpose in this study to understand the potential of digital technology in outdoors mathematics from the perspective of pre-service teachers.

THEORETICAL FRAMEWORK

One of the core ideas of this paper is that of Math Trail. Hence, is pertinent to begin by delimiting this concept. We consider a math trail to be a sequence of tasks along a pre-

planned route (with beginning and end), composed of a set of stops in which students solve mathematical tasks in the environment that surrounds them (Vale, Barbosa & Cabrita, 2019, adapted from Cross, 1997). This is a context that offers rich learning experiences to the participants, with the advantage of enabling the exploration of mathematical concepts stated in the curricular guidelines, aspect that can be seen as an advantage in the teachers' perspective (e.g. Barbosa & Vale, 2018; Vale, Barbosa & Cabrita, 2019). By experiencing a math trail, the participants can use and apply mathematical knowledge learned in school and also mobilize informal daily life knowledge. Beyond this possibility there is a wide range of skills summoned by outdoor education like problem solving, critical thinking, collaboration, communication, reasoning or establishing connections. For all the stated reasons, we must consider that it is important to complement the work developed inside the classroom with experiences in the outdoors, allowing students to discover and interpret the world beyond those four walls and accepting that education can take place in different contexts (Kenderov et al., 2009).

In a math trail the participants come into contact with realistic problems that highlight the usefulness of mathematics, but more than that amplify the possibility of establishing connections between mathematics and reality. This feature can be a game changer in inducing positive attitudes towards this discipline (e.g. Bonotto, 2001; Borromeo-Ferri, 2010), relying specially on curiosity, motivation and interest. Beyond solving realistic problems, in this context we must not forget the influence produced by movement in students' attitudes. Thinking and learning 'are not just in the head'; on the contrary, the body plays a decisive role in the entire intellectual process, from the first to the last years of our lives. Students who move, either in the classroom or in the outdoors, can learn, regardless of their activity, more effectively than those in typically sedentary classrooms (Hannaford, 2005). Alongside cognitive engagement, math trails bring into the table two other dimensions: physical and social engagement (Hannaford, 2005). The interaction between these dimensions, facilitated by a math trail, is in line with active learning, known by committing students to the learning process, hence promoting positive attitudes towards mathematics (e.g. Vale & Barbosa, 2018).

Nowadays, mobile devices are fully integrated in our daily lives and, consequently, in the lives of students from very young ages. Teachers should be more aware of this fact and try to keep up with this trend incorporating resources of this nature into their practices. In addition to following the development and needs of contemporary society, it is also important to state that mobile devices are becoming a tool with great potential in both classrooms and outdoor learning, enhancing students' learning and allowing the access to important information in different places and in a more aesthetic manner (Sung, Chang & Liu, 2016). In general, the influence of technology and the immediate availability of information inevitably have been recently shaping the ideas and skills to be developed by students as we move along the 21st century. Moving beyond the mere use of technology to a perspective of integration with other areas of knowledge, the STEM fields (Science, Technology, Engineering and Mathematics) are being highlighted, both in curricula and in literature, as an interconnected approach that brings opportunities for students to be engaged in an active learning perspective, solving realistic problems (NCTM, 2014, 2018). Refocusing the discussion on mobile devices, it is important to state that the diversity of learning opportunities offered by this type of technology (e.g. portability, allowing immediate learning and quick access to information, motivation, facilitating

communication between the teacher and the students) can make STEM education more interesting and enjoyable for students, widening the possibilities for engagement in STEM subjects, inside but also outside the classroom (e.g. Kärkkäinen & Vincent-Lancrin, 2013). The extension of the classroom to the outdoors is facilitated by the portability and wireless functionality of the mobile devices, which presents students with a more authentic and appropriate context (Cahyono & Ludwig, 2019). Digital technology can help develop a deeper understanding of mathematics, acting as a mind tool that facilitates inquiry, decision making, reflection, reasoning, problem solving and collaboration (Fessakis, Karta & Kozas, 2018).

METHODOLOGY

This is an on-going study that follows an interpretative qualitative methodology (Erickson, 1996). The participants are forty-eight students of an undergraduate teacher training course in primary education (6-12 years old). These pre-service teachers attend a unit course on Mathematics Education that acts as the context for the development of the study. Knowing that, so far, the participants did not have significant experiences working mathematics outside the classroom, we chose to start with an activity of this nature. Initially they completed a questionnaire (Questionnaire I) that aimed to access their perceptions about the teaching and learning of mathematics outside the classroom and also about the use of technology in that type of context. Then they had the opportunity to do a math trail using Math City Map (MCM), which was designed by the researchers to be used in the historical centre of the city of Viana do Castelo, Portugal. The pre-service teachers worked in groups of 3 and 4. They attributed the responsibility of the use of the app/smartphone to one of the elements of the group, while the others were in charge of the measurements, calculations and registers. After doing the trail they completed a second questionnaire (Questionnaire II), applied with the purposed to analyse eventual changes on the perceptions of the participants about outdoor mathematics and the use of technology, specifically the MCM app.

Data was collected in a holistic, descriptive and interpretive manner and included observations (of the pre-service teachers doing the math trail), questionnaires, photographs and written productions (solutions of the tasks). The latter were not used in this specific study. The researchers accompanied the participants during the trail, a fact that facilitated the accomplishment of the observation, allowing the access to reactions, comments, questions and attitudes. Since we had forty-eight participants, to maximize the observation, we chose to divide the group in half and do the math trail with each group separately. The questionnaires contained mainly open-ended questions, so that the content analysis focused on finding categories of responses regarding the perceptions evidenced by the participants, which were crossed with the evidences collected through the observation.

MAIN RESULTS AND DISCUSSION

Starting by analysing the results of Questionnaire I, we concluded that 91% of the participants considered that it is possible to teach and learn mathematics outside the classroom. The examples presented varied between: tasks related to real life situations; counting activities; money related tasks; shopping activities; games; competitions; clubs;

field trips; observing architecture/artwork/shapes in the outdoors; finding mathematics in nature, like patterns/shapes; doing a trail/peddy paper. 87% of the participants revealed that they have never had a mathematics class outdoors, which in a certain way may explain the general and vague ideas they had about how to do it. So, this fact indicates that, in order to incorporate certain methodologies in their future practices, pre-service teachers have to experience them first. As for technology knowledge, 60% of the participants stated that they did not know any digital resources to explore mathematics outdoors. The 40% that admitted knowing resources used for this purpose mentioned digital games, apps and robots, but none of the examples given allowed the exploration of the surrounding environment, they only had a playful strand.

Before going to the city centre to do the math trail with MCM, the participants had a brief session about the use of Math City Map. They came into contact with the main features of the app, to get acquainted before the activity, and downloaded the trail to the smartphones. Then the researchers accompanied them to the location of the trail and supervised the activity, which facilitated the observation of certain aspects. Regarding the use of the app, we can say that they didn't show noteworthy difficulties. They found it to be very intuitive and were extremely autonomous throughout the trail. The gamification feature was an extra motivating factor: on one hand it caused excitement when the solution was correct; and implied greater care before the introduction of the answers, which was reflected on several situations where the participants tried to make sure of the validity of the answer discussing it within their group. The dynamics of the math trail using MCM naturally promoted collaborative work, in each group, dividing responsibilities (e.g. smartphone; measurement; recording data; calculations), or even among different groups cooperating with the same goal in mind.

Throughout the mathematical trail, participants went through iconic points of the city of Viana do Castelo, having the opportunity to solve tasks centered on different mathematical contents (e.g. geometric transformations, patterns, measurements, estimates, areas, volumes, direct proportionality, visualization, counting). This dynamic allowed them to use a different lens, exploring the environment through a mathematical eye, but, at the same time, they could know better the city where they live in, observing more closely the elements directly worked on. In figures 1 and 2 we can observe different moments of the trail implementation that illustrate the pre-service teachers' work.









Figure 1: Pre-service teachers using Math City Map.







Figure 2: Pre-service teachers using Math City Map.

Throughout the trail it was possible to witness reactions and comments from the preservice teachers that we think are relevant and must be emphasized because they reveal engagement: the trail gave them the opportunity to get to know better certain aspects of the city, related to historical and architectural features that they did not know of; many expressed interest in using the app with their future students; we identified a generalized satisfaction throughout the activity; they valued the need to move around, opposed to the sedentary work traditionally developed inside the classroom.

After experiencing the math trail with MCM, the pre-service teachers filled Questionnaire II. From the analysis of the results we were able to conclude that all the participants recognized the importance of teaching and learning outside the classroom, especially as a way to complement the formal educational context. Contrary to the results obtained through Questionnaire I, they were all convinced, with no exception, that teaching and learning mathematics outside the classroom is possible, showing that some of these preservice teachers changed their opinion about this issue. Those who already though that this strategy was a possibility, stated it with even more emphasis, admitting that the experience exceed their expectations. We found several arguments supporting these ideas: follows the principles of active learning, promoting intellectual, social and physical engagement; learning is more meaningful for the students because they are directly involved; increases motivation and enthusiasm; helps understand the usefulness of mathematics, realizing its application in real life problems; allows to increase the knowledge of the cultural and natural heritage; facilitates collaborative work and helps develop communication skills, as well as critical thinking; it can lead to the use of technology.

The majority of these pre-service teachers expressed that they liked to solve all of the tasks presented along the trail, which is consistent with the observed motivation and enthusiasm. The tasks pointed as favourites corresponded to those considered to be the most challenging or the ones that presented information/curiosities/historical aspects about certain elements of the city that they did no know about. On the other hand, the least favourites were the ones that involved too many steps during the solution process, which led them to make some mistakes or to find the task to be too exhausting.

In this questionnaire the participants also commented on the use of MCM and its features. From the users/students perspective they highlighted as potentialities: the possibility to use curricular contents in real life situations; being user friendly and easy to understand, promoting autonomy; facilitating cooperation; helps to get to know the local environment;

develops spatial orientation; being more practical and interactive than the paper version; the possibility of getting immediate feedback; and the gamification feature. As for the teachers' perspective, the participants mentioned as potentialities: the possibility to design tasks adapted to the local environment and publishing them; addressing different mathematical contents and promoting interdisciplinary tasks; a way to diversify educational contexts; allows the teacher to supervise and accompany the work developed by the groups, due to the autonomy it provides the user. As limitations of the app, these pre-service teachers only referred to the possible lack of access to Wi-Fi, the fact that students of younger ages normally don't have smartphones and, in terms of the tasks, the limitation of the answer formats to either a value or multiple choice.

CONCLUSION

Based on previous studies developed with pre-service teachers (e.g. Barbosa & Vale, 2016; Barbosa & Vale, 2018; Vale & Barbosa, 2018; Vale, Barbosa & Cabrita, 2019) we had already concluded that designing and implementing math trails can promote positive attitudes towards mathematics and help gain a broader view of the connections we may establish with the surrounding environment. Math trails make mathematics come alive engaging the participants cognitively, emotionally and physically, which is why they can be associated with active learning. This type of experience enhances the "mathematical eye" of the trail designers as well as of the trail users (e.g. Vale, Barbosa & Cabrita, 2019), bringing out the usefulness and applications of mathematics.

This study focused only on the perspective of the trail user and not the designer. We intended to understand the potential of the MCM app in outdoor education from the point of view of pre-service teachers. Globally they valued the math trail experience as a meaningful pathway to engage students in realistic problem solving, that presents a diversity of opportunities for the establishment of connections between mathematics and other content areas, as well as with real life (e.g. Bonotto, 2001; Borromeo-Ferri, 2010). Active learning was also pointed out by the participants as a fundamental trait in a math trail, allowing intellectual, physical and social engagement, whose interaction normally generates positive attitudes (e.g. Hannaford, 2005; Vale & Barbosa, 2018). Math City Map was used as the means to present and execute the trail. This was the additional dimension of this study, trying to perceive its impact. These pre-service teachers valued the use of the app, finding it user friendly and motivating, especially due to the gamification feature. They also mentioned as positive its contribution for developing spatial orientation (moving with the help of the GPS and needing to recognize their position in space), cooperation (through group work and task division), students' autonomy and being more practical and interactive than the paper version. The only limitations recognized by the participants were related to constraints like the possible absence of Wi-Fi or smartphones (for example when working with students of younger ages) and also the limited possibilities for answer formats.

To conclude, when implementing the math trail there was certainly an additional motivation associated to the digital and interactive features of the MCM app, which facilitated and made more interesting the exploration of the outdoors from a mathematical point of view (e.g. Cahyono & Ludwig, 2019). Being pre-service teachers, the participants other than going through this experience as users, they also had the opportunity to assess

the potential of the strategy (math trail) and the resource (MCM app) and envision how they could, as teachers, implement it in the future. Recognizing the importance of keeping up with the technological development and society requirements they considered the possibility of integrating this resource, and the math trail strategy, in their practices.

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